

Measuring the Phase of Spatially Coherent Polychromatic Fields

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Realistic optical fields are never completely monochromatic and the interpretation of the concept of phase is not straightforward. It has been pointed out by Wolf that, if a strict condition of spatial coherence is imposed, the statistically averaged behavior of a polychromatic field can be described by an associated monochromatic wave. We show that, for spatially coherent polychromatic optical fields, the measurement of the phase of the second-order correlations determines the phase of this associated, spatially coherent field. We verify this prediction using a novel interferometric technique for measuring the cross-spectral density of a steady-state optical field.

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The phase of a steady-state optical field is usually discussed in terms of monochromatic waves [1]. However, it has been pointed out recently [2] that monochromaticity is not a necessary condition for generating sharp interference fringes and that it is, rather, a restriction on the spatial coherence that is required. Moreover, when a polychromatic wave field is spatially coherent at a given frequency ω , the correlation between light fluctuations at two different points factorizes in the form

$$W_{12}(\omega) = \mathcal{U}^*(P_1, \omega)\mathcal{U}(P_2, \omega), \quad (1)$$

where \mathcal{U} mimics an “equivalent” monochromatic field [3]. This fact is of considerable significance because it follows, in particular, that the phase of an optical wave field can be determined from correlation measurements. In this Letter, we prove that this equivalent field \mathcal{U} can be completely determined from measurements of the cross-spectral density: its amplitude from the square root of the spectral density and its phase (apart from a constant) from the phase of the spectral degree of coherence. For a situation that is well known in optics—the complex field in the focal plane of a convergent lens—we show that the phase Φ_W of the cross-spectral density $W_{12}(\omega)$ can also be regarded as the phase of a monochromatic field which would be focused by the same lens.

We have developed a novel interferometric technique that permits high sensitivity measurements of the spectral properties of light over narrow spatial regions. The experiment is schematically illustrated in Fig. 1 and more details can be found in [4]. Light from a broad band source is coupled into a fiber optic-based Michelson interferometer. In the reference arm, the light is collimated and reflected off a scanning mirror. In the measurement arm, the beam is collimated and expanded by an afocal system and it is subsequently focused by a lens L . In order to achieve the spatial resolution in the focusing region, the focal plane Σ_M is scanned by a small metallic sphere S of diameter $d = 0.75$ mm, which represents the second, pointlike mirror of the interferometer. The metallic sphere acts as a narrow band-pass filter for

the spatial frequencies in the Fourier plane of the lens L . The fields from the reference and test arm are then combined to give an interference signal at the detector. In the first step of the experiment, the pointlike mirror is placed at the point P_2 situated at a small distance ρ from the origin P_1 . Because the lengths of the two optical paths are such that the reference field scattered off the scanning mirror is equivalent with the retroreflection from the pointlike mirror situated at the point P_1 , the detected interference can be regarded as being in fact the interference between the light vibrations at two different points P_1 and P_2 in the plane Σ_M . During the measurement, the reference mirror is oscillated with constant speed introducing a periodic optical delay between the two interfering fields from the test and reference arms; the time and frequency analysis of the interference signal is synchronized with its motion. The intensity of the detected time-varying interference term is proportional to the real part of the time correlation between the light vibrations at P_1

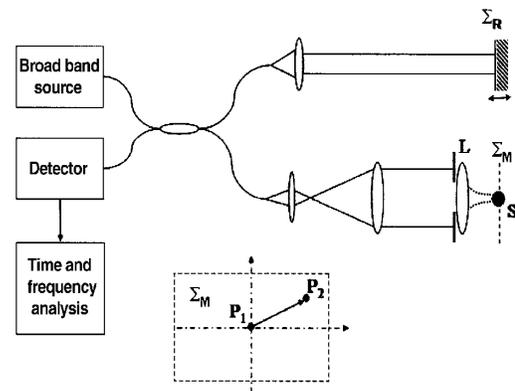


FIG. 1. Light from a polychromatic light source is launched in an interferometer using monomode optical fibers. Time and frequency analysis is performed for the interference between the plane wave retroreflected in the reference arm from an oscillating mirror Σ_R and the field backscattered by a pointlike mirror which is scanned in the plane Σ_M . The points P_1 and P_2 represent two subsequent positions of the pointlike mirror in the plane Σ_M .

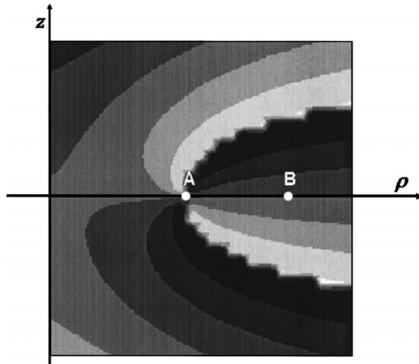


FIG. 2. Lines of equal phase in the neighborhood of the second Airy ring in the focal plane of a lens. The phase dislocation is clearly seen at the point A. The calculations are for the case of a quasimonochromatic Gaussian beam with the waist of 8.5 mm which is truncated by a circular aperture of 3.5 mm in diameter and focused by a lens with focal length 16 mm.

and P_2 and, according to the generalized Wiener-Khinchine theorem, is the Fourier transform of the cross-spectral density $W_{12}(\omega)$. In a second step of the experiment, the pointlike mirror is now placed at the point P_1 in the origin of the plane Σ_M ; a similar analysis of the time-varying interference provides the spectral density $S(\omega) = W_{11}(\omega)$ at the point P_1 . With this spatially resolved Fourier interferometer, we have been able to measure for the first time the complex cross-spectral density of a focused optical field [4].

If a quasimonochromatic field U of mean frequency ω is focused by a lens such as the one in our experiment, the dependence between the fields at P_1 and P_2 can be conveniently represented in terms of a complex field modifier M via the relation

$$U(\rho, \omega) = U(0, \omega)|M(0, \rho, \omega)| \exp[i\Phi_M(0, \rho, \omega)], \quad (2)$$

which can be evaluated using the generalized Debye theory for focusing applied to our experimental situation. Calculations were performed for the case of a Gaussian beam that is truncated by an aperture and the contours of the phase distribution in the neighborhood of the second Airy ring are shown in Fig. 2. The phase dislocation known to occur here is clearly seen at the point A.

In Fig. 3, we plot the phase Φ_W of the cross-spectral density which was measured for two different values of ρ in the points indicated by A and B in Fig. 2. The corresponding values of Φ_M evaluated using Debye's theory are indicated by continuous lines. As can be seen, the expected phase jump of π is found for the spectral phase measured close to A, correctly reproducing the phase dislocation which is characteristic for an Airy ring. For the scan B, on the other hand, the phase has been found to be wavelength independent as predicted by the calculations for the coherent field. These results show clearly that the phase Φ_W of the cross-spectral density can also be

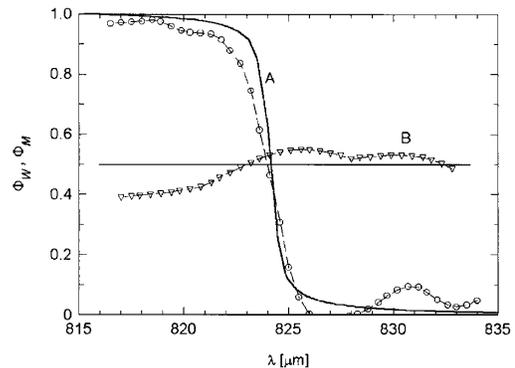


FIG. 3. Phase of the cross-spectral density measured in the points A and B of Fig. 2 situated at distances ρ equal to 76 and 96 μm , respectively. The symbols indicate the values of the measured Φ_W , while the continuous lines represent the phase Φ_M evaluated using the generalized Debye theory for an associated coherent field.

regarded as being the phase Φ_M of an associated monochromatic field, thus verifying the prediction in [2].

We stress that the phase of a spatially fully coherent field has been determined by measurements of second-order field correlations and that the approach is not limited to deterministic fields. Characterizing the phase of polychromatic fields via measurements of cross-spectral density can be particularly appealing in the case of random wave fields—speckle patterns—that result from the interaction between coherent or partially coherent waves and inhomogeneous media such as rough surfaces or multiple scattering volumes [4]. Understanding the rich behavior of the optical phase should be of paramount interest in the practice of imaging and scattering techniques dealing with the interaction between light and heterogeneous matter.

It is worth mentioning that in applications including microscopy, lithography, and data storage, which rely on properties of optical fields structures at subwavelength scales, the full spatial and spectral characterization of the field amplitude and phase must overcome significant challenges that are usually out of the reach for conventional techniques. The experiment described here opens up new possibilities for interferometric characterization of such finely structured optical fields.

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- [1] In this connection, see, for instance, K. A. Nugent, D. Paganin, and T. E. Gureyev, *Phys. Today* **54**, No. 8, 27 (2001).
 - [2] E. Wolf, *Opt. Lett.* (to be published).
 - [3] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995), p. 206.
 - [4] G. Popescu and A. Dogariu, *Phys. Rev. Lett.* **88**, 183902 (2002).