– Homework 1 –

ECE 564

(1) We found that the monochromatic plane wave is an <u>eigen function</u> of any linear system (operating on complex-valued functions).

Consider the linear system defined by the operator

$$\mathcal{L} = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

What is the <u>physical interpretation</u> of $e^{-i(\omega t - \bar{k}\cdot\bar{r})}$ being an eigen function of this operator?

(2) A laser has an optical spectrum consisting of two lines,

$$S(\omega) = A_1 e^{-\frac{(\omega-\omega_1)^2}{2\Delta\omega_1^2}} + A_2 e^{-\frac{(\omega-\omega_2)^2}{2\Delta\omega_2^2}}$$

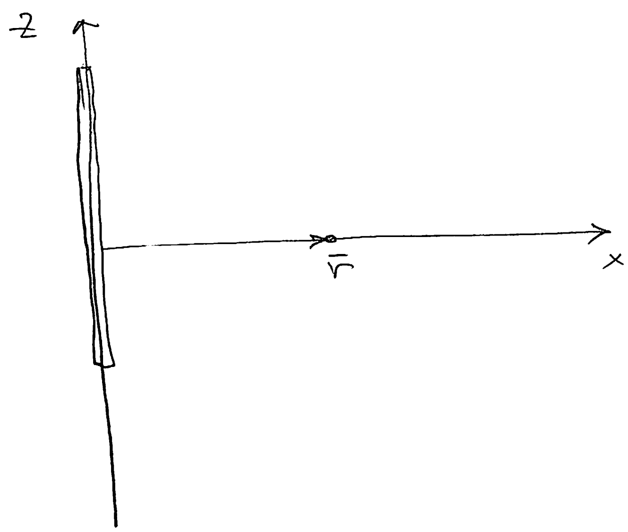
$$= S_1(\omega) + S_2(\omega)$$

a) Write down the autocorrelation function of this field.

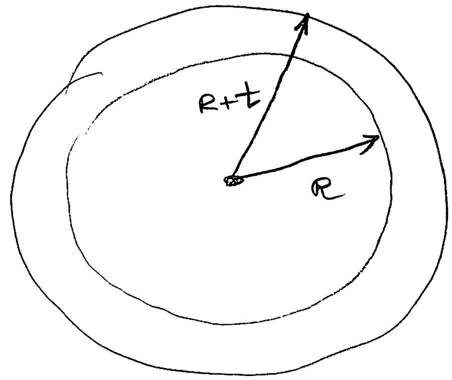
b) Write down the autocorrelation function if the spectrum is $S(\omega) = S_1(\omega) \circledast S_2(\omega)$

c) Calculate the autocorrelation function if $S(\omega) = S_1(\omega)\cdot S_2(\omega)$

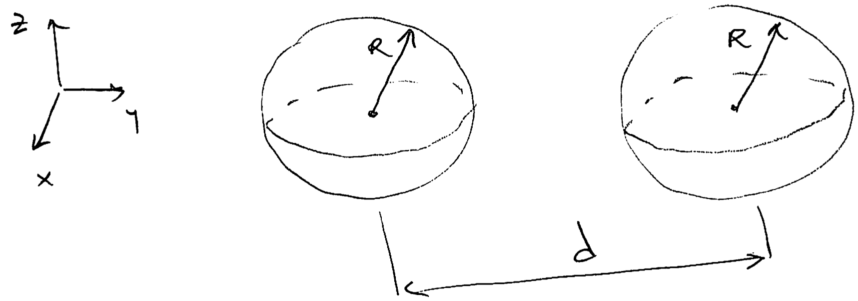
(3) Cylindrical waves: find the field at a point $\bar{r}$ emitted by an infinitely long source that is very thin: assume a source term $S(x,y,z;t) = S_0\,\delta(x,y)\,\delta(t)$.



(4). Calculate the Fourier transform of a ring of inner radius R and thickness t.



(5). Calculate the Fourier transform of a function represented by two ball functions, shifted by distance d.



(6). Calculate the Fourier transform of a function represented by a spherical shell of inner radius R and thickness t.

(7) A function is described by a ball function multiplied by a complex exponential, $f(\bar{r}) = e^{i k_0 z}\, \Pi\left(\frac{r}{2R}\right)$.

a) ~~note~~ write down the Fourier transform, $\tilde{f}(\bar{k})$.

b) calculate the 1st and 2nd moments of $|\tilde{f}(\bar{k})|^2$.

3) Discuss the physical significance of this problem.