Chapter 2 - Properties of Light

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2.1 Properties of EM Fields

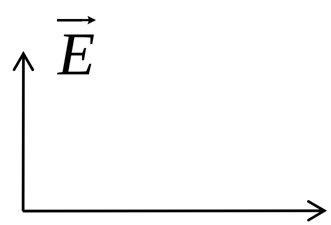
- Amplitude A and phase ϕ are random functions in both time and space:

$$\vec{E}(\vec{r},t) = \vec{e}A(\vec{r},t).e^{i\phi(\vec{r},t)} \qquad (1)$$

2.1 Properties of EM Fields

a) Polarization:

- Gives the direction of field oscillation
- Generally, light is a transverse wave (unlike sound = longitudinal)



Propagation ($\bar{k}$ = wave vector)

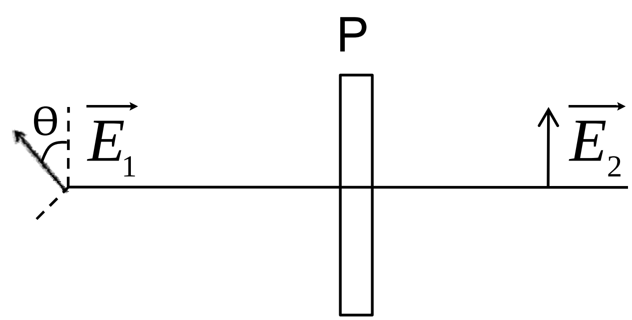
$$|\bar{k}| = \frac{2\pi}{\lambda}$$

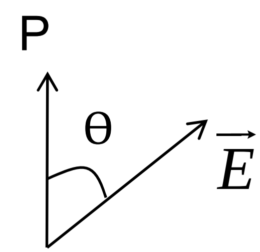
- Anisotropic materials: different optical properties along different axis → useful

2.1 Properties of EM Fields

a) Polarization:

- There is always a basis $\left(\hat{x}, \hat{y}\right)$ for decomposing the field into 2 polarizations (eigen modes); equivalently (right, left) circular polarization is also a basis.
- Dichroism: different absorption for different pol → one way to create polarizers:



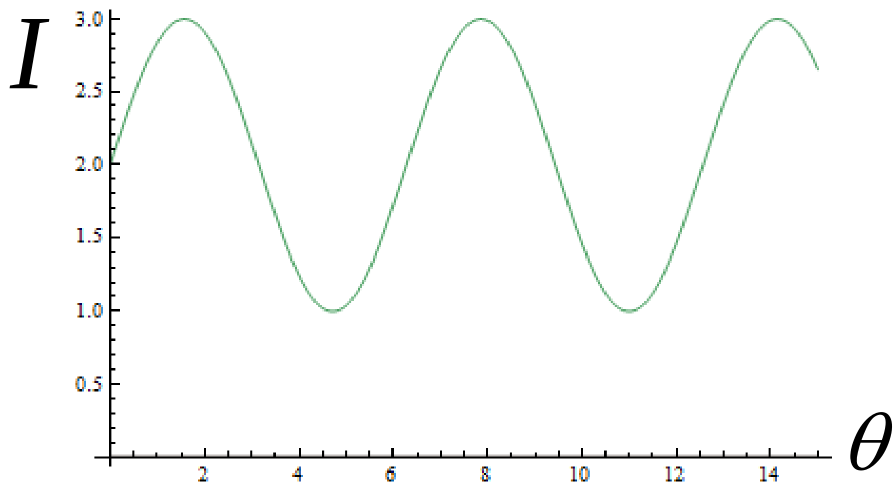


- Malus Law: $\left|E_2\right| = \left|E_1\right| * \cos\theta$ (2)

2.1 Properties of EM Fields

a) Polarization:

$$I = |E|^2 ; I_2 = I_1 * \cos^2 \theta$$



Malus Law

- Birefringence – Different refr. index for different pol.

2.1 Properties of EM Fields

a) Polarization:

- Natural Light → unpolarized → superposition $E_x = E_y$ with no phase relationship between the two
- Circularly polarized → $E_x = E_y$, $\phi_x - \phi_y = \pi/2$!
- Matrix formalism of polarization transformation

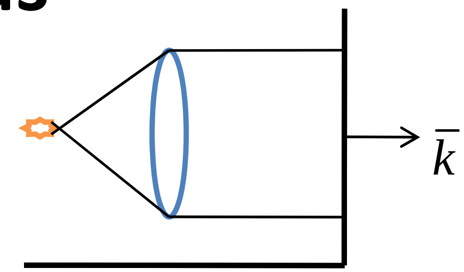
  (Jones – 2x2, complex & Muller – 4x4, real)

  We'll do this later.

$$\begin{pmatrix} E_x' \\ E_y' \end{pmatrix} = J \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

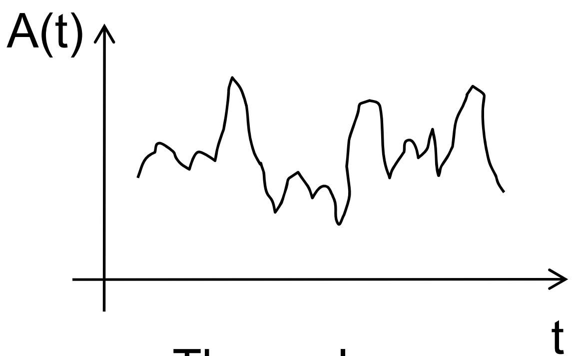
$$|E|^2 = I \rightarrow \text{Stokes Vect. Dim } 4 \qquad J_{ij} \in \mathbb{C}$$

2.1 Properties of EM Fields

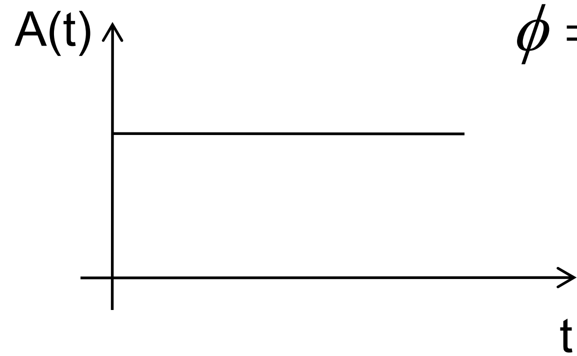


b) Amplitude: $\left[A(\vec{r},t)\right]=\frac{V}{m}$

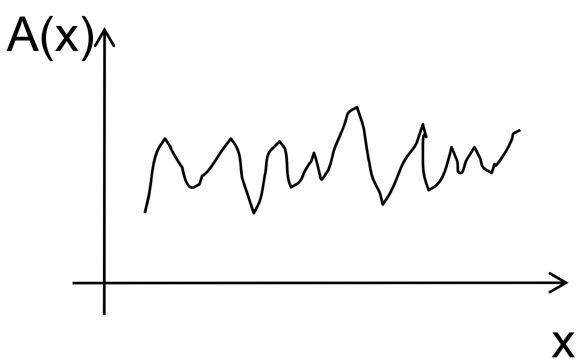
$\phi = k\lambda$



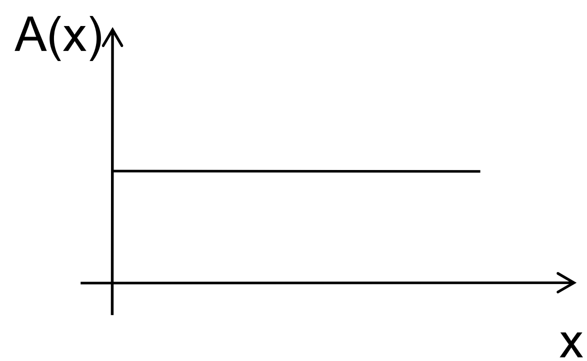
•Thermal source



•Stabilized laser

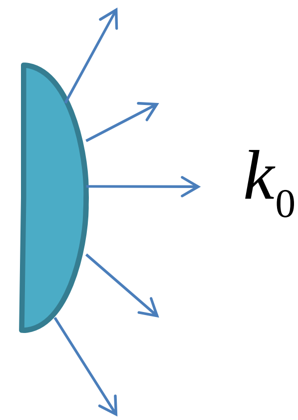


•Arbitrary field

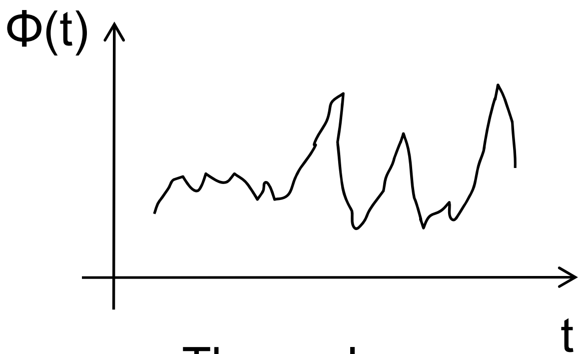


•Plane Wave

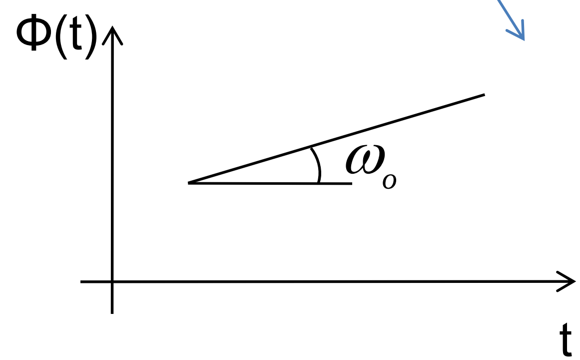
2.1 Properties of EM Fields



c) Phase: $[\Phi]$ = rad

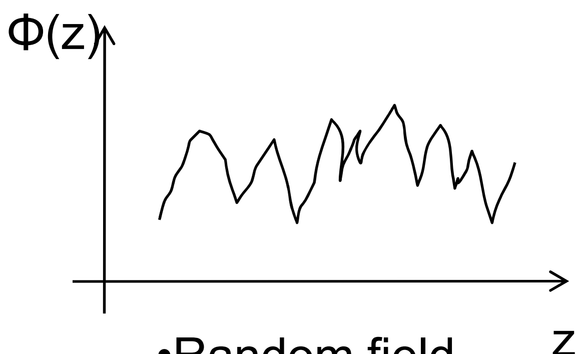


•Thermal source

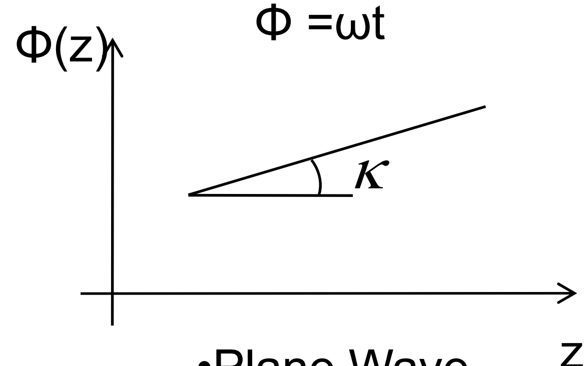


•Laser at freq ω_o

$\Phi = \omega t$



•Random field



•Plane Wave

$\Phi = kz$

2.1 Properties of EM Fields

c) Phase: [Φ] = rad

- For quasi-monochromatic fields, plane wave

$$\phi = \omega t - \vec{k} \cdot \vec{r}$$

- $k = \dfrac{\omega}{c} = \dfrac{2\pi\upsilon}{c} = \dfrac{2\pi}{Tc} = \dfrac{2\pi}{\lambda} =$ wave number (3)

$$\lambda = cT; \; T = \frac{1}{N}; \; \omega = 2\pi N$$

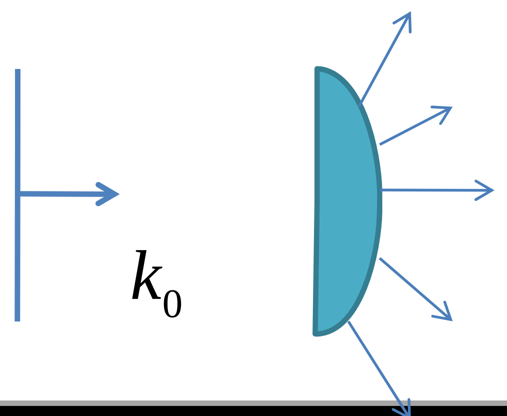
2.2 The frequency domain representation

- Random variable E(t) has a frequency-domain counterpart:

$$E(\omega) = A(\omega)e^{i\phi(\omega)} \qquad (4)$$

- Similarly E(x) has a frequency-domain pair:

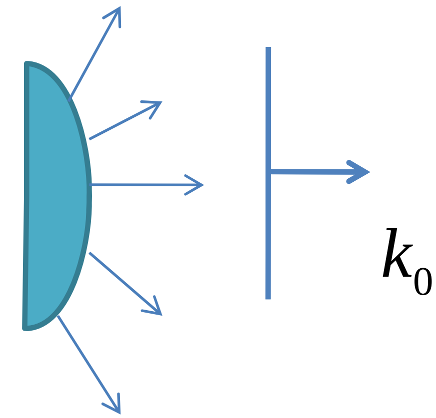
$$E(\xi) = A(\xi)e^{i\phi(\xi)} \qquad (5)$$



$$\phi(\omega) = k \cdot z = n(\omega) \cdot k_0 \cdot z$$

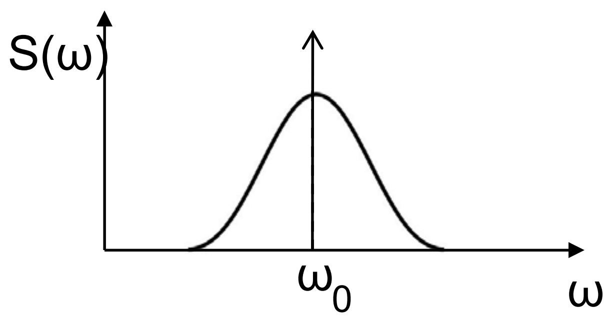
$$k_0 = \frac{2\pi}{\lambda}$$

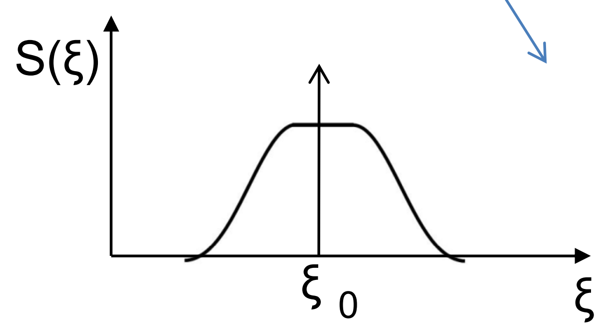
2.2 The frequency domain representation



a) Spectral amplitude:

- Optical Spectrum: $S(\omega)=\left|A(\omega)\right|^2$
- Angular Spectrum: $S(\xi)=\left|A(\xi)\right|^2$





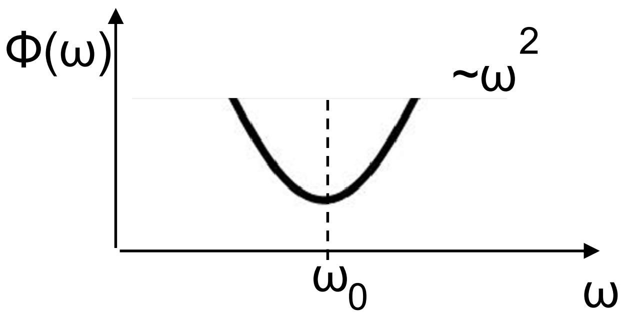
- $\left[\xi\right]=m^{-1}$ = <u>Spatial Frequency</u> (connects to angular spectrum)
- Tipically: t $\leftrightarrow$ ω ; x - ξ } Will follow similar equations
- The information contained is the same (t, ω) and (x,ξ)

2.2 The frequency domain representation

b) Spectral phase:

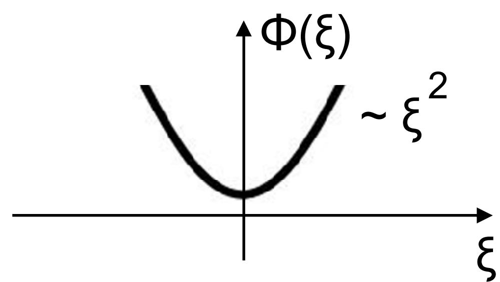
- Phase delay of each spectral component

Optical Frequency $\alpha = chirp$



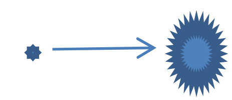
•Dispersive material
(linear chirp)

Spatial Frequency



•Defocused point source
(1st order aberration)

A point is mapped to a blur



- Full similarity between (t,ω) and (x,ξ)

$$\frac{d}{d\omega}\omega^2 \sim \omega$$

2.3 Measurable Quantities

- The information about the system under investigation may be contained in polarization and:
  - A(t), ϕ(t)
  - A(ω), ϕ(ω)

  } (t,ω)

  - A(x), ϕ(x)
  - A(ξ), ϕ(ξ)

  } (x,ξ)

  } 8 quantities

- Experimentally, we have access only to:

$$I = \left\langle \left| A(t) \right|^2 \right\rangle = \text{time average}$$

2.3 Measurable Quantities

- Experimentally, we have access only to:

$$I = \left\langle \left| A(t) \right|^2 \right\rangle = \text{time average} \qquad (6)$$

- i.e the phtodetectors (photodiode, CCD, retina, etc) produce photoelectrons:

$$h\upsilon = E_{e^-} + W \quad \text{(Einstein)} \qquad (7)$$

$h\upsilon$ → Photon incident energy

E_{e^-} → Electron kinetic energy

W → Work

2.3 Measurable Quantities

- All detectors sensitive to power/energy
- However, all 8 quantities can be accessed via various tricks
- Eg1: Want I(λ) → measure I(θ) and use a device with $\theta(\lambda)$
- Eg2: Want ϕ → use interferometry → I(ϕ)$\alpha \left|E_1\right|\left|E_2\right|\cos(\phi_1 - \phi_2)$

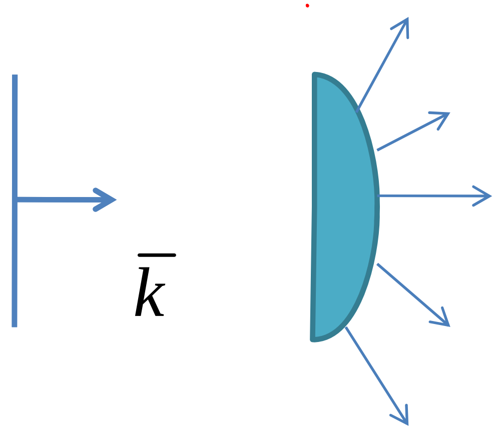
2.4 Uncertainty Principle

- Space - momentum or energy-time cannot be measured simmultaneously with infinite accuracy

$$\begin{cases} \Delta\overline{x} \cdot \Delta\overline{p} = \text{constant} \simeq h \simeq \text{Plank's constant} \\ \Delta E \Delta t = \text{constant} \end{cases}$$

- For photons:

$$\begin{cases} E = \hbar\omega \\ \overline{p} = \hbar\overline{k};\ p = \dfrac{h}{\lambda} \end{cases}$$



2.4 Uncertainty Principle

a) t $-\omega$

$$\hbar \Delta\omega \Delta t = \text{constant}$$

$$\rightarrow \boxed{\Delta\omega \Delta t \simeq 2\pi}$$

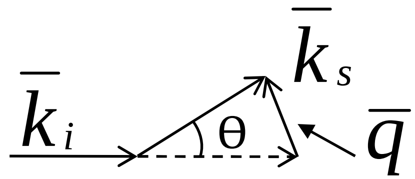
- Implications:

  1- short pulses require broad spectrum

  2-high spectral resolution requires long time of measurement

2.4 Uncertainty Principle

b) $x - \xi$



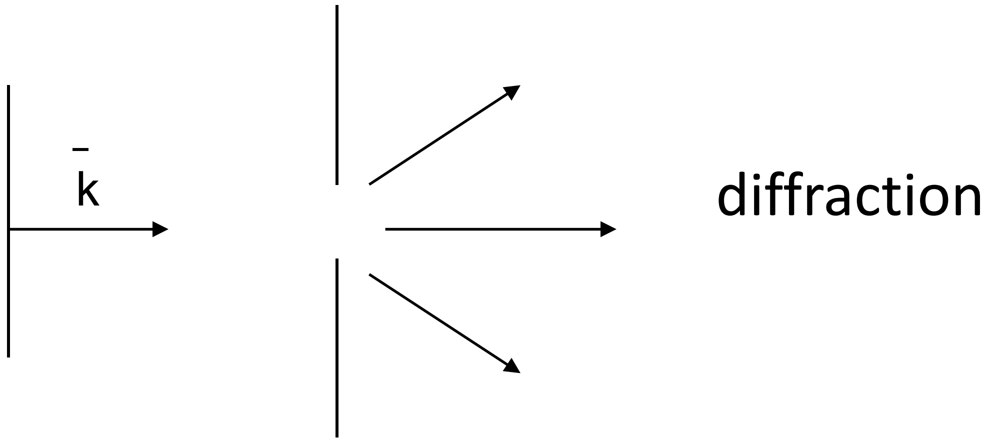
$$\overline{\Delta p} = h(\bar{k}_s - \bar{k}_i) = h\bar{q}$$

$$\rightarrow \boxed{\Delta x \left|\bar{q}\right| \simeq \pi} \quad ; \quad \left|\bar{q}\right| = 2k\sin\left(\frac{\theta}{2}\right)$$

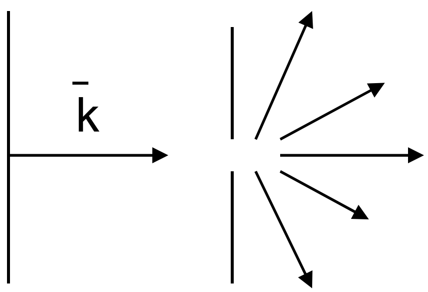
$$\rightarrow \quad \Delta x \frac{2\sin(\theta/2)}{\lambda} \simeq 1 \quad ; \quad \boxed{\theta \sim \frac{\lambda}{\Delta x}}$$

$$\rightarrow \boxed{\Delta x_{\min} \approx \frac{\lambda}{2}}$$ - meaning of resolution

2.4 Uncertainty Principle



- Smaller aperture → Higher angles



- If aperture < $\frac{\lambda}{2}$, light doesn't go through (easily)
- <u>Eg</u>: Microwave door

2.4 Uncertainty Principle

- We will encounter these relationships many times later
- Fourier may have understood this uncertainity principle way before Heisenberg!