Chapter 2 - Properties of Light

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2.1 Properties of EM Fields

- Amplitude $A$ and phase $\phi$ are random functions in both time and space:

$$\vec{E}(\vec{r}, t) = e A(\vec{r}, t).e^{i\phi(\vec{r}, t)}$$ (1)
2.1 Properties of EM Fields

a) Polarization:
   - Gives the direction of field oscillation
   - Generally, light is a transverse wave (unlike sound = longitudinal)

\[ \vec{k} = \text{wave vector} \]

\[ |\vec{k}| = \frac{2\pi}{\lambda} \]

- Anisotropic materials: different optical properties along different axis → useful
2.1 Properties of EM Fields

a) Polarization:
   - There is always a basis \((\hat{x}, \hat{y})\) for decomposing the field into 2 polarizations (eigen modes); equivalently (right, left) circular polarization is also a basis.
   - Dichroism: different absorption for different pol \(\rightarrow\) one way to create polarizers:

\[
\begin{align*}
\theta; \quad & \vec{E}_1 \\
\vec{E}_2 & = |E_1| \cos \theta
\end{align*}
\]

\[
\begin{align*}
P & \quad \vec{E}_1 \\
\vec{E}_2 & \quad \vec{E}
\end{align*}
\]

- Malus Law: \( |E_2| = |E_1| \cos \theta \) (2)
2.1 Properties of EM Fields

a) Polarization:

\[ I = |E|^2 ; I_2 = I_1 \times \cos^2 \theta \]

- Birefringence – Different refr. index for different pol.
2.1 Properties of EM Fields

a) Polarization:
   - Natural Light $\rightarrow$ unpolarized $\rightarrow$ superposition $E_x = E_y$ with no phase relationship between the two
   - Circularly polarized $\rightarrow$ $E_x = E_y$, $\phi_x - \phi_y = \pi/2$
   - Matrix formalism of polarization transformation
     (Jones – 2x2, complex & Muller – 4x4, real)

We’ll do this later.

$$\begin{pmatrix} E_x' \\ E_y' \end{pmatrix} = J \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$|E|^2 = I \rightarrow$ Stokes Vect. Dim 4

$J_{ij} \in \mathbb{C}$
2.1 Properties of EM Fields

b) Amplitude: \[ A(r,t) \] = \frac{V}{m}

- Thermal source
- Arbitrary field
- Stabilized laser
- Plane Wave

\[ \phi = k \lambda \]
2.1 Properties of EM Fields

c) Phase: \[ \Phi = \text{rad} \]

- Thermal source
  \[ \Phi(t) \]

- Random field
  \[ \Phi(z) \]

- Laser at freq \( \omega_0 \)
  \[ \Phi(t) \]

- Plane Wave
  \[ \Phi = kz \]

\[ k_0 \]
2.1 Properties of EM Fields

c) Phase: [\Phi] = \text{rad}

- For quasi-monochromatic fields, plane wave

\[ \phi = \omega t - \vec{k} \cdot \vec{r} \]

- \[ k = \frac{\omega}{c} = \frac{2\pi
u}{c} = \frac{2\pi}{Tc} = \frac{2\pi}{\lambda} = \text{wave number} \quad (3) \]

\[ \lambda = c T ; \quad T = \frac{1}{N} ; \quad \omega = 2\pi N \]
2.2 The frequency domain representation

- Random variable $E(t)$ has a frequency-domain counterpart:

$$E(\omega) = A(\omega)e^{i\phi(\omega)} \quad (4)$$

- Similarly $E(x)$ has a frequency-domain pair:

$$E(\xi) = A(\xi)e^{i\phi(\xi)} \quad (5)$$

\[ \phi(\omega) = k \cdot z = n(\omega) \cdot k_0 \cdot z \]

\[ k_0 = \frac{2\pi}{\lambda} \]
2.2 The frequency domain representation

a) Spectral amplitude:
   - Optical Spectrum: \( S(\omega) = |A(\omega)|^2 \)
   - Angular Spectrum: \( S(\xi) = |A(\xi)|^2 \)

\[
\begin{align*}
\left[ \xi \right] &= m^{-1} = \text{Spatial Frequency} \quad \text{(connects to angular spectrum)} \\
\text{Tipically: } t &\leftrightarrow \omega \\
x &- \xi \quad \text{Will follow similar equations} \\
\text{The information contained is the same } (t, \omega) \text{ and } (x, \xi)
\end{align*}
\]
2.2 The frequency domain representation

b) Spectral phase:
   - Phase delay of each spectral component

   Optical Frequency: $\alpha = \text{chirp}$
   
   Spatial Frequency: $\sim \xi^2$

   - Dispersive material (linear chirp)
   - Defocused point source (1st order aberration)

   - Full similarity between $(t, \omega)$ and $(x, \xi)$

   $\frac{d}{d\omega} \omega^2 \sim \omega$
2.3 Measurable Quantities

- The information about the system under investigation may be contained in polarization and:
  - \( A(t), \phi(t) \) (t,\( \omega \))
  - \( A(\omega), \phi(\omega) \)
  - 8 quantities
  - \( A(x), \phi(x) \) (x,\( \xi \))
  - \( A(\xi), \phi(\xi) \)

- Experimentally, we have access only to:
  \[
  I = \left\langle |A(t)|^2 \right\rangle = \text{time average}
  \]
2.3 Measurable Quantities

- Experimentally, we have access only to:
  \[ I = \left\langle |A(t)|^2 \right\rangle = \text{time average} \quad (6) \]

- i.e., the photodetectors (photodiode, CCD, retina, etc) produce photoelectrons:

  \[ h\nu = E_{e^-} + W \quad \text{(Einstein)} \quad (7) \]

  - Photon incident energy
  - Electron kinetic energy
  - Work
2.3 Measurable Quantities

- All detectors sensitive to power/energy
- However, all 8 quantities can be accessed via various tricks
  - **Eg1**: Want $I(\lambda) \rightarrow$ measure $I(\theta)$ and use a device with $\theta(\lambda)$
  - **Eg2**: Want $\phi \rightarrow$ use interferometry $\rightarrow I(\phi) \propto |E_1||E_2|\cos(\phi_1 - \phi_2)$
2.4 Uncertainty Principle

- Space - momentum or energy-time cannot be measured simultaneously with infinite accuracy

\[
\Delta x \cdot \Delta p = \text{constant} \cong h \cong \text{Plank's constant}
\]
\[
\Delta E \Delta t = \text{constant}
\]

- For photons:

\[
\begin{cases}
E = \hbar \omega \\
p = \hbar k; \quad p = \frac{\hbar}{\lambda}
\end{cases}
\]
2.4 Uncertainty Principle

a) \( t - \omega \)

\[ \hbar \Delta \omega \Delta t = \text{constant} \]

\[ \Rightarrow \Delta \omega \Delta t \approx 2\pi \]

- Implications:
  1. short pulses require broad spectrum
  2. high spectral resolution requires long time of measurement
2.4 Uncertainty Principle

b) $x - \xi$

\[ \bar{k}_i \xrightarrow{\theta} \bar{k}_s \xrightarrow{\bar{q}} \]

\[ \Delta p = h(\bar{k}_s - \bar{k}_i) = h\bar{q} \]

\[ \begin{align*}
\Delta x |q| & \approx \pi \\
|q| & = 2k \sin \left(\frac{\theta}{2}\right)
\end{align*} \]

\[ \Delta x \frac{2\sin(\theta / 2)}{\lambda} \approx 1 \ ; \ \theta \sim \frac{\lambda}{\Delta x} \]

\[ \Delta x_{\text{min}} \approx \frac{\lambda}{2} \ - \ \text{meaning of resolution} \]
2.4 Uncertainty Principle

- Smaller aperture $\rightarrow$ Higher angles

- If aperture $< \frac{\lambda}{2}$, light doesn’t go through (easily)
- Eg: Microwave door
2.4 Uncertainty Principle

- We will encounter these relationships many times later
- Fourier may have understood this uncertainty principle way before Heisenberg!