Lecture 3
Radiometric and Photometric Properties of Light
Photon-based Radiometric Quantities
Number of Photons

• Each photon carries an energy $hv$ or $hc/\lambda$.

• Total energy of optical

\[ Q = N hv \]
\[ N = \frac{Q}{hv}. \]

$N$— the number of photons (unitless)

• Define an entirely equivalent set of radiometric quantities
  • Valuable whenever discrete nature of light is relevant.
  • Cases: studying photodetection
Photon Density

- Photon density $n$
  - The number of photons per unit volume.
  \[
  n = \frac{N}{V} \quad \text{[n]} = m^{-3}
  \]

Photon Flux

- Photon flux $P_q$
  - The photon quantity analogous to power.
  \[
  P_q = \frac{dN}{dt} \quad \text{[P}_q\text{]} = s^{-1}
  \]
  $q$ — a photon quantity
  - Connection between $P_q$ and $P$ (power) is straightforward.
  \[
  P = \frac{dQ}{dt} = \frac{d(N \cdot h\nu)}{dt} = h\nu \frac{dN}{dt} = h\nu P_q
  \]
Photon Temporal Power Spectrum

• Photon temporal power spectrum $S_q$
  • The photon flux per unit temporal frequency.

$$S_q(v) = \frac{dP_q(v)}{dv} \quad [S_q] = s^{-1} / \text{Hz}$$

$S_q(v)$ — the number of photons per second in the infinitesimal interval $(v, v+dv)$

• Energy-based power spectrum $S(v)$

$$S_q(v) = \frac{d}{dv} \left[ \frac{P(v)}{hv} \right] = \frac{1}{hv} \left[ S(v) - \frac{P(v)}{v} \right]$$

For $\omega = 2\pi v$, $S_q(\omega) = \frac{1}{2\pi} S_q(v)$

For $\lambda = c/v$, $S_q(\lambda) = -\frac{c}{\lambda^2} S_q(v)$
Photon Intensity

- Photon intensity $I^q_\Omega$
  - The photon flux per unit solid angle, $\Omega$.

$$I^q_\Omega = \frac{dP^q_\Omega (\Omega)}{d\Omega} \quad [I^q_\Omega] = \text{s}^{-1} / \text{srad}$$

$$d\Omega = \frac{\hat{n} \cdot dA}{r^2} = \frac{dk_x dk_y}{k^2}$$

$$I^q_\Omega = k^2 \frac{d^2 P^q_\perp (\mathbf{k}_\perp)}{dk^2_\perp} = k^2 S^q_\perp (\mathbf{k}_\perp)$$

$$d^2 \mathbf{k}_\perp = dk_x dk_y$$

$S^q_\perp (\mathbf{k}_\perp) — \text{the spatial power spectrum of the photon flux}$
Photon Irradiance

• Photon irradiance $I_q$
  • The photon flux per unit area.
  \[
  I_q = \frac{dP_q}{dA} \hspace{1cm} [I_q] = \frac{s}{m^2}
  \]

• Energy-based irradiance
  \[
  I = \frac{dP}{dA} = h\nu \frac{dP_q}{dA} = h\nu I_q
  \]

Photon Spectral Irradiance

• Photon spectral irradiance $I_v^q$
  • The photon flux per unit area, per unit of optical frequency.
  \[
  I_v^q = \frac{d^2P_q}{dAd\nu} \hspace{1cm} [I_v^q] = \frac{s}{m^2 Hz}
  \]
Photon Radiance

- Photon radiance $L_q$
  - The photon flux emitted or absorbed by a surface, per unit solid angle, per unit projected area.

$$L_q = \frac{d^2 P_q(\Omega, A)}{d\Omega dA \cos \theta} \quad [L_q] = \text{s}^{-1} / \text{srad} \cdot \text{m}^2$$

$d\cos \theta$ — the projection of the area normal to the direction of interest

Photon Spectral Radiance

- Photon spectral radiance $L_q^q$
  - The photon radiance per unit frequency.

$$L_q^q = \frac{dL_q}{d\nu} \quad [L_q^q] = \frac{s^{-1}}{m^2 \text{srad} \cdot \text{Hz}}$$
Photon Exitance

• Photon exitance $M_q$
  • The amount of photon flux emitted by a source per unit area.

$$M_q = \frac{dP_q}{dA} \quad [M_q] = \frac{s^{-1}}{m^2}$$

• **Note:** Photon exitance describes a source, while the photon irradiance is a property of the field and shares the same units.

Photon Spectral Exitance

• Photon spectral exitance $M^q_v$
  • The photon exitance per unit frequency.

$$M^q_v = \frac{dM^q_v}{d\nu} \quad [M^q_v] = \frac{s^{-1}}{m^2 Hz}$$
Photometric Properties of Light
Photometric properties of light

• Photometry
  • Properties of the light perceived by the human eye (subjective photodetector).
  • Spectral sensitivity of the eye is taken into account.

• Luminosity function
  • The eye spectral response
    • Day-time (photopic) adaptation
    • Dark time (scotopic) adaptation

• Day time spectral response — Luminous

Figure 4.1. Photopic (daytime-adapted, green curve) and scotopic (darkness-adapted blue curve) luminosity functions.
Luminous Energy

• Luminous energy $Q_l$
  • The total energy weighted according to the luminosity function.
  • Measured in lumen-second. Lumen is the unit for luminous flux.

$$[Q_l] = \text{lm} \cdot \text{s}$$

Luminous Flux

• Luminous flux (or luminous power) $P_l$
  • The photometric equivalent of power.
  • Weighted by the luminosity.

$$P_l = \frac{dQ_l}{dt} \quad [P_l] = \text{lm}$$
Luminous Energy Density

- Luminous energy density \( W_l \)
  - The luminous energy per unit volume
    \[
    W_l = \frac{dQ_l}{dV} \quad [W_l] = \frac{lm \cdot s}{m^3}
    \]

Luminous Intensity

- Luminous intensity \( I_l^\Omega \)
  - The luminous flux per unit solid angle
    \[
    I_l^\Omega = \frac{dP_l}{d\Omega} \quad [I_l^\Omega] = \frac{lm}{srad} = Cd\text{(candela)}
    \]

- **Note**: Candela (cd) is a primary units comprising the International System of Units (SI).
4.4. Luminous Intensity

• Standard of candela
  • The candela is the luminous intensity in a given direction, of a source that emits monochromatic radiation of frequency $540 \times 10^{12} \text{ Hz}$ and that has a radiant intensity in that direction of $1/683 \text{ W/rad}$.
    • For the frequency of $540 \times 10^{12} \text{ Hz}$, $\lambda=c/\nu=555 \text{ nm}$, at which the human eye is the most sensitive in bright conditions.

• The luminous intensity with respect to wavelength
  
  \[ I_t^\Omega(\lambda) = [I_\Omega(\lambda)Y(\lambda)]683 \text{ lm/W} \]

  $I_\Omega$ — radiometric intensity (in W/srad)
  $Y(\lambda)$ — the photopic luminosity function evaluated at the particular wavelength

\[ \text{Wavelength [nm]} \]
\[ \text{Luminosity function} \]

\[ \text{Figure 4.1. Photopic (daytime-adapted, green curve) and scotopic (darkness-adapted blue curve) luminosity functions.} \]
Illuminance

- Illuminance \( I_l \) (Equivalent to the photometric irradiance)
  - The luminous flux per unit area.
  \[
  I_l = \frac{dP_l}{dA}
  \]
  \([I_l] = \frac{lm}{m^2} = \text{lux, lx}\)

Luminance

- Luminance \( L_l \) (Equivalent to the photometric radiance)
  - The luminous power per solid angle, per normal unit area of the source.
  \[
  L_l = \frac{d^2 P_l}{d\Omega dA \cos(\theta)}
  \]
  \([L_l] = \frac{lm}{\text{srad} \cdot m^2} = \frac{cd}{m^2}\)
  - Brightness is sometimes used interchangeably with luminance.