Chapter 9 – Electro-Optics

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Electro-Optics

- 1st order effect:

$$P_i^{w} = -\frac{1}{\varepsilon_0}\varepsilon_{ii}\varepsilon_{jj} r_{ijk} E_j(\omega) E_k(0)$$

DC

110_z

$$D_i = \varepsilon_0 n_i^2 E_i - \underbrace{\varepsilon_0 n_i^2 n_j^2 r_{ijk} E_j E_k}(DC)$$

$$\Rightarrow \begin{cases} Dx = \varepsilon_0 n_0^2 - \varepsilon_0 n_0^4 r_{123} E_y E_{Dc} - \varepsilon_0 n_0^2 n_e r_{133} E_z E_{DC} \\ Dy = \varepsilon_0 n_0^2 - \varepsilon_0 n_0^4 r_{213} E_x E_{Dc} - 0 \\ Dz = \varepsilon_0 n_e^2 E_z \end{cases}$$

$$r_{53}^{kDP} \equiv 0$$

By using the contracted indices (7.1-11), the equation of the index ellipsoid in the presence of an electric field can be written

$$\left(\frac{1}{n_x^2} + r_{1k}E_k\right)x^2 + \left(\frac{1}{n_y^2} + r_{2k}E_k\right)y^2 + \left(\frac{1}{n_z^2} + r_{3k}E_k\right)z^2 \qquad (7.2\text{-}3)$$
$$+ 2yzr_{4k}E_k + 2zxr_{5k}E_k + 2xyr_{6k}E_k = 0$$

where E_k $(k = 1, 2, 3)$ is a component of the applied electric field and summation over repeated indices k is assumed. Here $1, 2, 3$ correspond to the principal dielectric axes x, y, z, and n_x, n_y, n_z are the principal refractive indices. This new ellipsoid (7.2-3) reduces to the unperturbed ellipsoid (7.1-1) when $E_k = 0$. In general, the principal axes of the ellipsoid (7.2-3) do not coincide with the unperturbed axes (x, y, z).

A new set of principal axes can always be found by a coordinate rotation, which is known as the principal-axis transformation of a quadratic form. The dimensions and orientation of the ellipsoid (7.2-3) are, of course, dependent on the direction of the applied field as well as the 18 matrix elements r_{ik}. We have argued above that in crystals possessing an inversion symmetry (centrosymmetric), $r_{ik} = 0$. The form, but not the magnitude, of the tensor r_{ik} can be derived from symmetry considerations, which dictate which of the 18 coefficients r_{ik} are zero, as well as the relationships that exist between the remaining coefficients. In Table 7.2 we give the form of the electro-optic tensor for all the noncentrosymmetric crystal classes. The electro-optic coefficients of some crystals are listed in Table 7.3.

7.2.1. Example: The Electro-optic Effect in KH_2PO_4

Consider the specific example of a crystal of potassium dihydrogen phosphate (KH_2PO_4), also known as KDP. The crystal has a fourfold axis of symmetry, which by strict convention is taken as the z (optic) axis, as well as two mutually orthogonal twofold axes of symmetry that lie in the plane normal to z. These are designated as the x and y axes. The symmetry group of this crystal is $\bar{4}2$m. Using Table 7.2, we write the electro-optic tensor in the form

$$r_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}, \qquad (7.2\text{-}4)$$

Table 7.2. Electro-optic Coefficients in Contracted Notation for All Crystal Symmetry Classes[a]

Centrosymmetric ($\bar{1}$, 2/m, mmm, 4/m, 4/mmm, $\bar{3}$, $\bar{3}$m 6/m, 6/mmm, m3, m3m):

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Triclinic:

1

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix}$$

Monoclinic:

2 $(2 \parallel x_2)$

$$\begin{pmatrix} 0 & r_{12} & 0 \\ 0 & r_{22} & 0 \\ 0 & r_{32} & 0 \\ r_{41} & 0 & r_{43} \\ 0 & r_{52} & 0 \\ r_{61} & 0 & r_{63} \end{pmatrix}$$

2 $(2 \parallel x_3)$

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ r_{41} & r_{42} & 0 \\ r_{51} & r_{52} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

m $(m \perp x_2)$

$$\begin{pmatrix} r_{11} & 0 & r_{13} \\ r_{21} & 0 & r_{23} \\ r_{31} & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & r_{53} \\ 0 & r_{62} & 0 \end{pmatrix}$$

m $(m \perp x_3)$

$$\begin{pmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ r_{31} & r_{32} & 0 \\ 0 & 0 & r_{43} \\ 0 & 0 & r_{53} \\ r_{61} & r_{62} & 0 \end{pmatrix}$$

Orthorhombic:

222

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

2mm

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Table 7.2. (*Continued*).

Tetragonal:

$$4: \begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \bar{4}: \begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & -r_{13} \\ 0 & 0 & 0 \\ r_{41} & -r_{51} & 0 \\ r_{51} & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} \qquad 422: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$4mm: \begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \bar{4}2m \ (2 \parallel x_1): \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

Trigonal:

$$3: \begin{pmatrix} r_{11} & -r_{22} & r_{13} \\ -r_{11} & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ -r_{22} & -r_{11} & 0 \end{pmatrix} \qquad 32: \begin{pmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}$$

$$3m \ (m \perp x_1): \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} \qquad 3m \ (m \perp x_2): \begin{pmatrix} r_{11} & 0 & r_{13} \\ -r_{11} & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}$$

Table 7.2. (*Continued*).

Hexagonal:

$$6: \begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad 6mm: \begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad 622: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\bar{6}: \begin{pmatrix} r_{11} & -r_{22} & 0 \\ -r_{11} & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & -r_{11} & 0 \end{pmatrix} \qquad \bar{6}m2 \ (m \perp x_1): \begin{pmatrix} 0 & -r_{22} & 0 \\ 0 & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} \qquad \bar{6}m2 \ (m \perp x_2): \begin{pmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}$$

Cubic:

$$\bar{4}3m, 23: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{pmatrix} \qquad 432: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

[a] The symbol over each matrix is the conventional symmetry-group designation.

so that the only nonvanishing elements are $r_{41} = r_{52}$ and r_{63}. Using Eqs. (7.2-3) and (7.2-4), we obtain the equation of the index ellipsoid in the presence of a field $\mathbf{E}(E_x, E_y, E_z)$ as

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41}E_x yz + 2r_{41}E_y xz + 2r_{63}E_z xy = 1, \quad (7.2\text{-}5)$$

where the constants involved in the first three terms do not depend on the field and, since the crystal is uniaxial, are taken as $n_x = n_y = n_o$, $n_z = n_e$. We thus find that the application of an electric field causes the appearance of "mixed" terms in the equation of the index ellipsoid. These are the terms with xy, xz, yz. This means that the major axes of the ellipsoid, with a field

Electro-Optics

$$\Rightarrow \varepsilon_{ij} = \varepsilon_0 \begin{pmatrix} n_0^2 & -n_0^4 r_{63} E_{Dc} & 0 \\ -n_0^4 r_{63} E_{Dc} & n_0^2 & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}$$

$$\varepsilon_{ij}' = W(-\beta)\overline{\overline{\varepsilon}} \cdot W(\beta) =$$

$$= \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} n_0^2 & -\Delta \\ -\Delta & n_0^2 \end{pmatrix} \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}$$

$$\beta = \frac{\pi}{4} \Rightarrow \xi_{ij} = \xi_0 \begin{pmatrix} n_0^2 + \Delta & 0 & 0 \\ 0 & n_0^2 - \Delta & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}; \quad \boxed{\Delta = n_0^4 r_{63} E_z(DC)}$$

b_i

biaxial crystal

Modulators

- Eg KDP(tetra $\bar{4}2m$) r_{41}, r_{52}, r_{63} only three nonzero elements

$$\begin{cases} n_x{}^2 = n_0{}^2 + \Delta; \\ n_x = \sqrt{n_0{}^2 + \Delta} = n_0\sqrt{1 + \dfrac{\Delta}{n_0{}^2}} \simeq n_0 + \dfrac{1}{2} n_0 \dfrac{\Delta}{n_0{}^2} = n_0 + \dfrac{1}{2}\Delta n_0 \\ \qquad = n_0 + \dfrac{1}{2} n_0{}^3 r_{63} E_z(DC) \\ n_y = n_0 - \dfrac{1}{2}\dfrac{\Delta}{r_{01}} \end{cases}$$

Modulators

$$\Gamma = \frac{2\pi}{\lambda}(n_x{}' - n_y{}')d = \frac{\Delta}{n_0} = \frac{2\pi}{\lambda_0} n_0{}^3 r_{63} \underbrace{E_z(DC)d}_{V}$$

$$\Gamma = \pi \Rightarrow \boxed{V_\pi = \frac{\lambda_0}{2 n_0{}^3 r_{63}}}$$

$$\boxed{T = \sin^2 \frac{\Gamma}{2}}$$

linearize

$$\text{Add } QW \Rightarrow T = \sin^2\left[\frac{\pi}{4} + \frac{\pi}{2}\frac{V}{V_\pi}\right]$$

Modulators

- Let $V = V_m \sin \omega_m t$

$$T = \sin^2\left[\frac{\pi}{4} + \frac{\pi}{2}\Gamma_m \sin \omega_m t\right] =$$

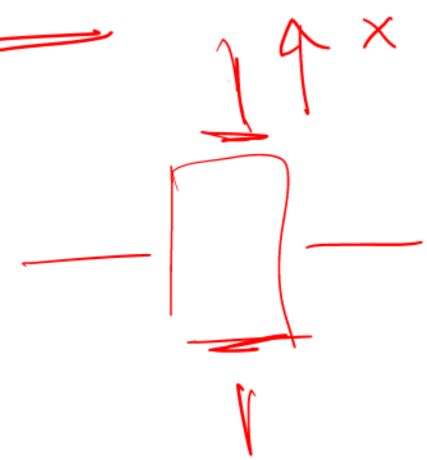
$$= \frac{1}{2}\left[1 - \cos\left[\frac{\pi}{2} + \Gamma_m \sin \omega_m t\right]\right] = \frac{1}{2}\left[1 + \sin\left[\Gamma_m \sin \omega_m t\right]\right]$$

linear

$$\Gamma_m \ll 1 \rightarrow T = \frac{1}{2}\left(1 + \Gamma_m \sin \omega_m t\right) \sim \Gamma_m$$

Quadratic (Kerr)

$$P_i^{(\omega)} = -\frac{1}{\varepsilon_0}\varepsilon_{ii}\varepsilon_{jj}s_{ijk\ell}E_j(\omega)E_k(DC)E_\ell(DC)$$



Applications of EO

- longitudinal
- transverse

} modulators

- For $LiNiO_3$:

$$\begin{cases} n_x = n_0 - \frac{1}{2} n_0{}^3 r_{13} E \\ n_y = n_0 - \frac{1}{2} n_0{}^3 r_{13} E \\ n_z = n_e - \frac{1}{2} n_e{}^3 r_{33} E \end{cases}$$

$$\phi = \underbrace{\frac{2\pi}{\lambda_0} n_0 L}_{\phi_0} - \frac{\pi}{\lambda v} n_0{}^3 r_{13} \underbrace{V}_{Ed}$$

$$V_\pi = \frac{\lambda v}{n_0{}^3 r_{13}} \Rightarrow$$ Phase mod(indep. of polariz.)

Chapter 9 – Acousto-optics

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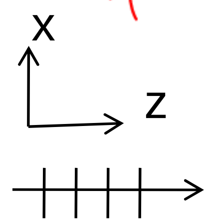
Acousto-optics

$$P_i = -\varepsilon_0 \sum_{jkl} n_j^{\,2} n_i^{\,2} p_{ijkl} S_{kl} E_j$$

12 → 6
13 → 5
23 → 4

- ac wave:



$$\Rightarrow S_{13} = S_5$$

$$\overline{U}(z,t) = \hat{x} A \cos(\Omega t - kz)$$

Table 9.1. Forms of the Elasto-optic Coefficients in Contracted Notation for all Classes of Crystal Symmetry

Triclinic (36)†

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} \\ p_{51} & p_{52} & p_{53} & p_{54} & p_{55} & p_{56} \\ p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & p_{66} \end{pmatrix}$$

Monoclinic (20)

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & p_{15} & 0 \\ p_{21} & p_{22} & p_{23} & 0 & p_{25} & 0 \\ p_{31} & p_{32} & p_{33} & 0 & p_{35} & 0 \\ 0 & 0 & 0 & p_{44} & 0 & p_{46} \\ p_{51} & p_{52} & p_{53} & 0 & p_{55} & 0 \\ 0 & 0 & 0 & p_{64} & 0 & p_{66} \end{pmatrix}$$

Orthorhombic (12)

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & 0 & 0 \\ p_{31} & p_{32} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{66} \end{pmatrix}$$

Tetragonal (10)
classes $4, \bar{4}, 4/m$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & p_{16} \\ p_{12} & p_{11} & p_{13} & 0 & 0 & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 \\ 0 & 0 & 0 & -p_{45} & p_{44} & 0 \\ p_{61} & -p_{61} & 0 & 0 & 0 & p_{66} \end{pmatrix}$$

Tetragonal (7)
classes $4mm, 422, 4/mmm$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{66} \end{pmatrix}$$

Trigonal (12)
classes $3, \bar{3}$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{12} & p_{11} & p_{13} & -p_{14} & -p_{15} & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ p_{41} & -p_{41} & 0 & p_{44} & p_{45} & -p_{51} \\ p_{51} & -p_{51} & 0 & -p_{45} & p_{44} & p_{41} \\ -p_{16} & p_{16} & 0 & -p_{15} & p_{14} & \frac{1}{2}(p_{11}-p_{12}) \end{pmatrix}$$

Table 9.1. (*Continued*).

Trigonal (8)
classes $3m, 32, \bar{3}m$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & 0 & 0 \\ p_{12} & p_{11} & p_{13} & -p_{14} & 0 & 0 \\ p_{13} & p_{13} & p_{33} & 0 & 0 & 0 \\ p_{41} & -p_{41} & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & p_{41} \\ 0 & 0 & 0 & 0 & p_{14} & \frac{1}{2}(p_{11}-p_{12}) \end{pmatrix}$$

Hexagonal (8)
classes $6, \bar{6}, 6/m$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & p_{16} \\ p_{12} & p_{11} & p_{13} & 0 & 0 & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 \\ 0 & 0 & 0 & -p_{45} & p_{44} & 0 \\ -p_{16} & p_{16} & 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) \end{pmatrix}$$

Hexagonal (6)
classes $\bar{6}m2, 6mm, 622, 6/mmm$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) \end{pmatrix}$$

Cubic (4)
classes $23, m3$

$$\begin{pmatrix} p_{11} & p_{12} & p_{21} & 0 & 0 & 0 \\ p_{21} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{21} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix}$$

Cubic (3)
classes $\bar{4}3m, 432, m3m$

$$\begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix}$$

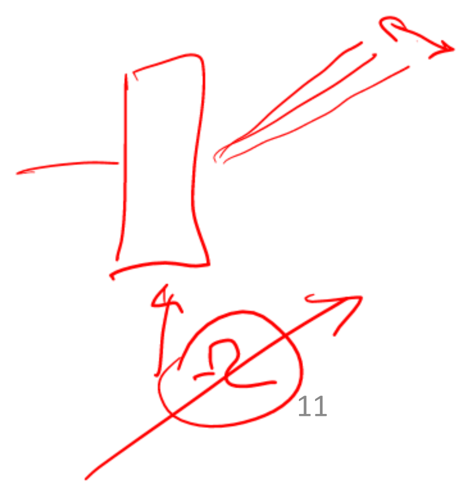
Table 9.2. Strain–Optic Coefficients[a] [1]

(a) Isotropic System

Substance	Wavelength λ (μm)	p_{11}	p_{12}
Fused silica (SiO_2)	0.63	0.121	0.270
As_2S_3 glass	1.15	0.308	0.299
Water	0.63	±0.31	±0.31
$Ge_{33}Se_{55}As_{12}$ (glass)	1.06	±0.21	±0.21
Lucite	0.63	±0.30	±0.28
Polystyrene	0.63	±0.30	±0.31

(b) Cubic System: Classes $\bar{4}3m$, 432, and m3m

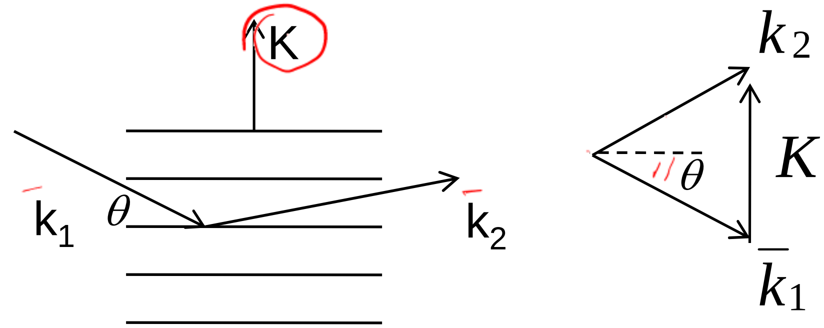
Substance	Wavelength λ (μm)	p_{11}	p_{12}	p_{44}	$p_{11} - p_{12}$
CdTe	10.60	−0.152	−0.017	−0.057	−0.135
GaAs	1.15	−0.165	−0.140	−0.072	−0.025
GaP	0.633	−0.151	−0.082	−0.074	−0.069
Ge	2.0–2.2	−0.063	−0.0535	−0.074	−0.0095
	10.60	0.27	0.235	0.125	
NaCl	0.55–0.65	0.115	0.159	−0.011	−0.042



Bragg regime

Acousto-optics

$K = \frac{\Omega}{v}$



$\bar{K} = \bar{k}_2 - \bar{k}_1$

$$2nk_0 \sin\theta = K$$

$$k_0 = 2\pi / \lambda$$

$$K = 2\pi / \Lambda$$

$$\sin\theta_B = \frac{\lambda}{2\Lambda}; \quad \textit{Bragg angle}$$

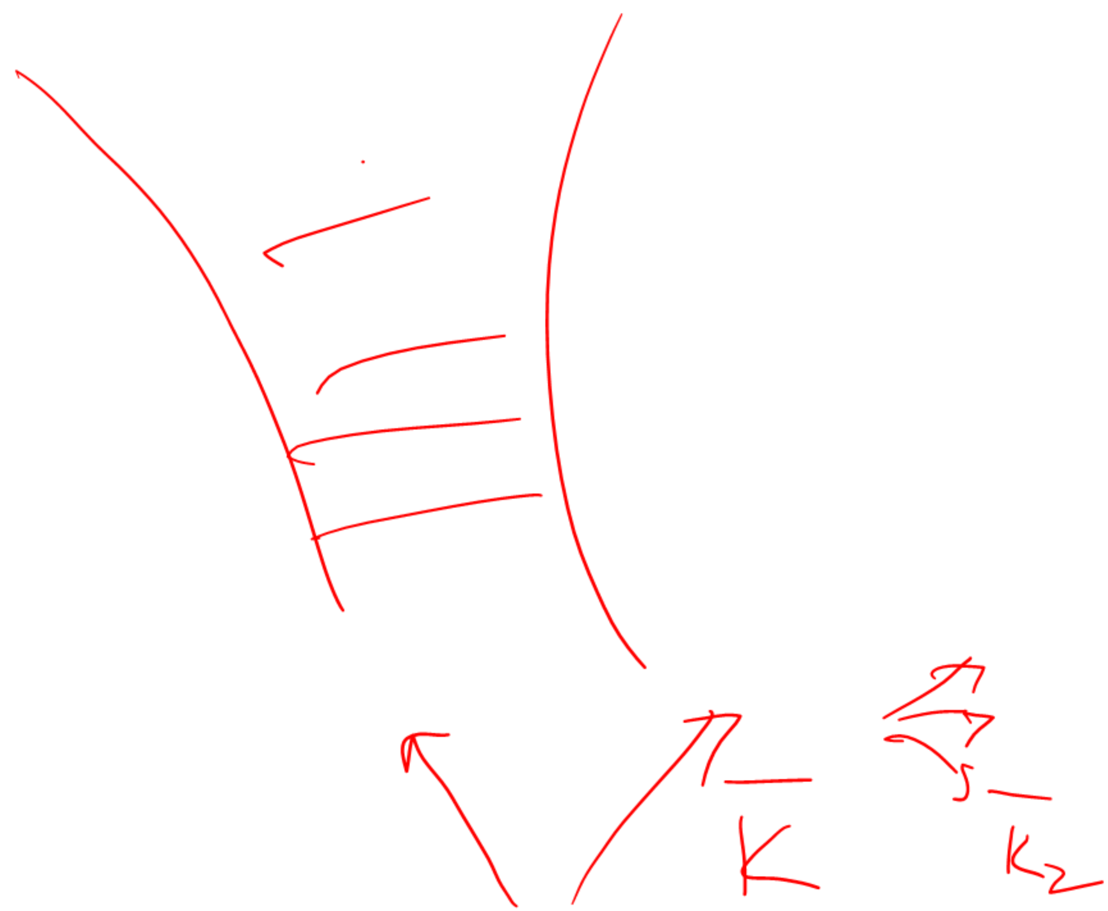
$\bar{k}_{1,2} = \frac{\omega}{c}\hat{k}_{1,2}$

$\bar{K} = \frac{\Omega}{v}\hat{K}$

$2nk_0 \sin\theta = \frac{4\pi n}{\lambda} \sin\theta = \frac{2\pi}{\Lambda}$

$\omega_2 = \omega_1 \pm \Omega$

Raman-Nath



Acousto-optics

$$\theta = small(|\bar{k}| \ll |k_0| \Leftrightarrow \Omega \ll \omega_0)$$

Doppler Shift

$$\begin{aligned}\Delta\omega &= \mathbf{\Delta k} \cdot \mathbf{\Delta v} \\ &= K v_s \\ &= \Omega\end{aligned}$$

$$\Delta\omega = \Omega$$

Quantum mechanics

$$\begin{cases} \bar{k}' = \bar{k} + \bar{\kappa} \\ \hbar\omega' = \hbar\omega + \hbar\Omega \end{cases}$$

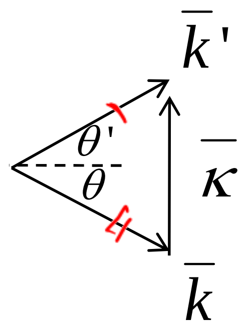
$\bar{k}' = \bar{k} + \bar{\kappa}$ —— conservation of momentum

$\hbar\omega' = \hbar\omega + \hbar\Omega$ —— conservation of energy

Anisotropic media

$$\bar{k}' - \bar{k} = \bar{\kappa}$$

n-different



θ - negative

$$k' \sin\theta' + k \sin\theta = \kappa; \theta \neq \theta'$$

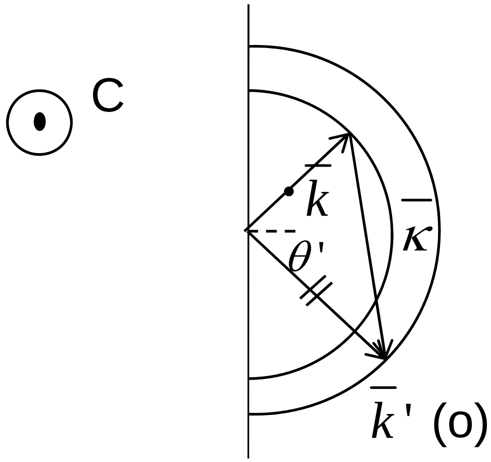
$$\frac{2\pi n'}{\lambda_0} \sin\theta' + \frac{2\pi n}{\lambda_0} \sin\theta = \frac{2\pi}{\Lambda}; \quad \kappa = \frac{2\pi}{\Lambda}$$

$$\sin\theta' = \frac{\lambda_0}{n'\Lambda} - \frac{n}{n'} \sin\theta$$

$\Lambda =$ wavelength of sound

Anisotropic

Ex:



$\bar{k}$ - (e)

$\bar{k}'$ - scattered in prop. Plane – (o)

$$\Rightarrow \sin\theta' = \frac{\lambda_0}{n'\Lambda} - \frac{n_e}{n_0}\sin\theta$$

$$\theta' = \frac{\pi}{2} \Rightarrow \theta = -\frac{\pi}{2} \quad \Rightarrow \quad \Lambda = \frac{\lambda_0}{n_0 - n_e} \gg \lambda_0$$

$$\theta' = \frac{\pi}{2}, \theta = \frac{\pi}{2} \quad \Rightarrow \quad \Lambda = \frac{\lambda_0}{n_0 + n_e}$$

$$\boxed{\frac{\lambda_0}{n_0 + n_e} < \Lambda < \frac{\lambda_0}{|n_0 - n_e|}}$$

Small angle Scattering

$$\frac{I_{scatt}}{I_{inc}} = \sin^2(\chi L)$$

$$\chi = \frac{k_0 (n_1 n_2)^{3/2}}{4\sqrt{\cos\theta_1 \cos\theta_2}} \hat{e}_i \overline{\overline{\overline{p}}}_{ijke} \overline{\overline{S}}_{ke} \hat{e}_j$$

Kin. Energy/ V = ½ W_{total}

$$I_{ac} = \frac{1}{2}\rho v_s \left|\frac{\partial U}{\partial t}\right|^2 = \frac{1}{2}\rho v_s \Omega^2 |\overline{u}|^2 = \frac{1}{2}\rho v_s{}^3 [\underbrace{\kappa |\vec{U}|}_{\overline{S}}]^2$$

$\Omega = \kappa v_s$

$$\overline{S}\left(\frac{\partial U}{\partial z} = \kappa U\right)$$

$$\Rightarrow \boxed{I_{ac} = \frac{1}{2}\rho v_s{}^3 \overline{S}^2}$$

Small angle Scattering

$$\overline{S} = \sqrt{\frac{2I_{ac}}{\rho v_s^3}} \Rightarrow \boxed{\chi = \frac{k_0 (n_1 n_2)^{3/2}}{4\sqrt{\cos\theta_1 \cos\theta_2}} \overline{P} \sqrt{\frac{2I_{ac}}{\rho v_s^3}}}$$

Small $\theta \Rightarrow$ cos→ 1

$$\boxed{M = \frac{n^6 \overline{p}^2}{\rho v_s^3}} \rightarrow \overline{p}$$

table

$$\Rightarrow \chi \cong \frac{k_0 (n_1 n_2)^{3/2}}{4} \sqrt{\frac{\rho v_s^3 M}{n^6} \frac{2I_{ac}}{\rho v_s^3}} = \boxed{\frac{\pi}{\sqrt{2}\lambda_0} \sqrt{M I_{ac}} = \chi}$$

Small angle Scattering

Detuning: $\sin\theta_B = \dfrac{\kappa}{2k}$

$$\sin\theta_2 = \sin\theta_1 + \frac{\kappa}{k}; \theta_2 = \theta_B - \Delta\theta$$

$$\Delta\beta = k_1 \sin\theta_1 - k_2 \sin\theta_2$$

$$(Bragg) = 2k\sin\theta = \kappa$$

$$\delta(\Delta\beta) = k_1 \cos\theta_1 \delta\theta_1 - k_2 \cos\theta_2 \delta\theta_2$$

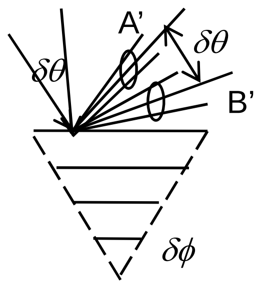
$$\delta\theta_1 = \delta\theta_2 = \delta\theta$$

$$\delta(\Delta\beta) = k(\cos\theta_1 - \cos\theta_2)\delta\theta =$$

$$= 2k\sin\theta_B \delta\theta = \kappa\delta\theta$$

$$\Rightarrow \frac{I_{scat}}{I_{inc}} = \frac{\chi^2}{\chi^2 + \left(\frac{1}{2}\kappa\Delta\theta\right)^2} \sin^2\left[\chi L \sqrt{1 + \left(\frac{\kappa\Delta\theta}{2\chi}\right)^2}\right]; \quad s^2 = \chi^2 + \left(\frac{1}{2}\kappa\Delta\theta\right)^2$$

Finite Beams



$$\delta\theta \cong \frac{2\lambda}{\pi n w_0}; \delta\phi = \frac{\Lambda}{L}$$

size of acoustic beam

$$\Delta\theta = \delta\theta + \delta\phi; \quad \boxed{\delta\phi = \delta\theta} \simeq \frac{1}{2}\Delta\theta; \delta\phi = \frac{\Lambda}{2L}$$

$$\Rightarrow \boxed{\Delta f_s = \frac{2nv_s\cos\theta}{\lambda_0}\frac{2\lambda_0}{\pi n w_0} = \frac{4v_s\cos\theta}{\pi w_0}} \text{ - Full } \Rightarrow \delta v_s = \frac{1}{2}$$

$$\delta f \sim \frac{1}{W_0}; \text{ or } (\delta\phi = \delta\theta) \to \Delta f = \frac{2nv_s\cos\theta}{\lambda_0}\frac{\Lambda}{L}$$

! Not overlap with undiffracted order

N spots

$$N = \frac{\Delta\theta}{\delta\theta} = \frac{\lambda_0 \Delta f}{2nv_s \cos\theta} \frac{\pi n W_0}{2\lambda_0} = \boxed{\underbrace{\left(\frac{\pi W_0}{2v_s}\right)}_{\tau} \Delta f = N}$$

$$\boxed{\Delta\theta_B = \frac{\lambda}{2nv_s}\Delta f} \quad \text{; cond } \delta\phi > \Delta\theta_B$$

$$\frac{\Lambda_0}{L} \geq \frac{\lambda_0}{2nv_s}\Delta f \Rightarrow \boxed{\frac{\Delta f}{f_0} \leq \frac{2n\Lambda^2}{\lambda_0 L}}$$