Aberration Theory

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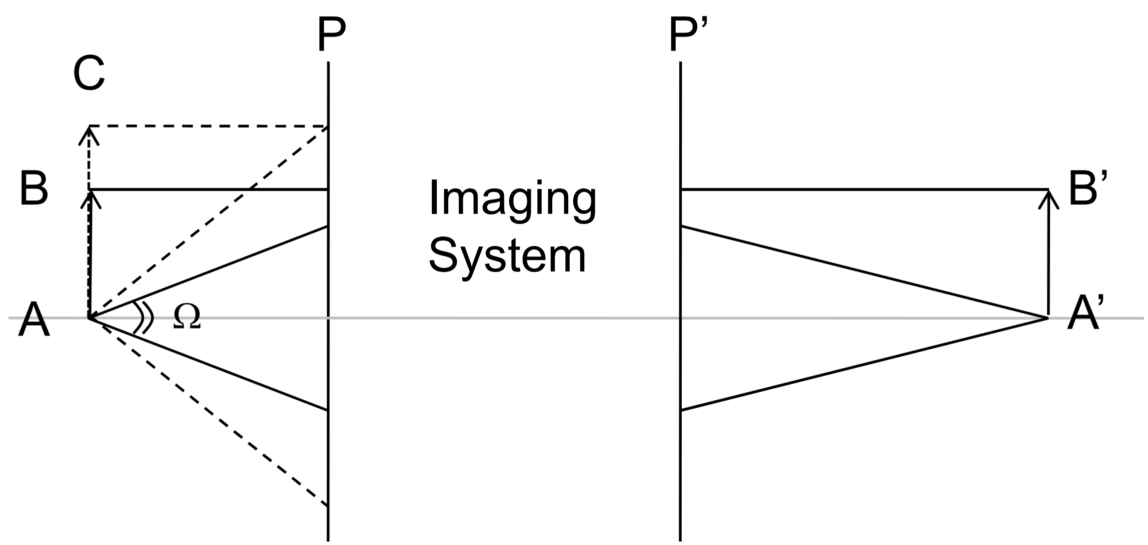
1. Pupils and Windows

- Consider a object imaged by the system below:



! The dotted lines do not make it through the system, i.e, they are blocked somewhere

1. Pupils and Windows

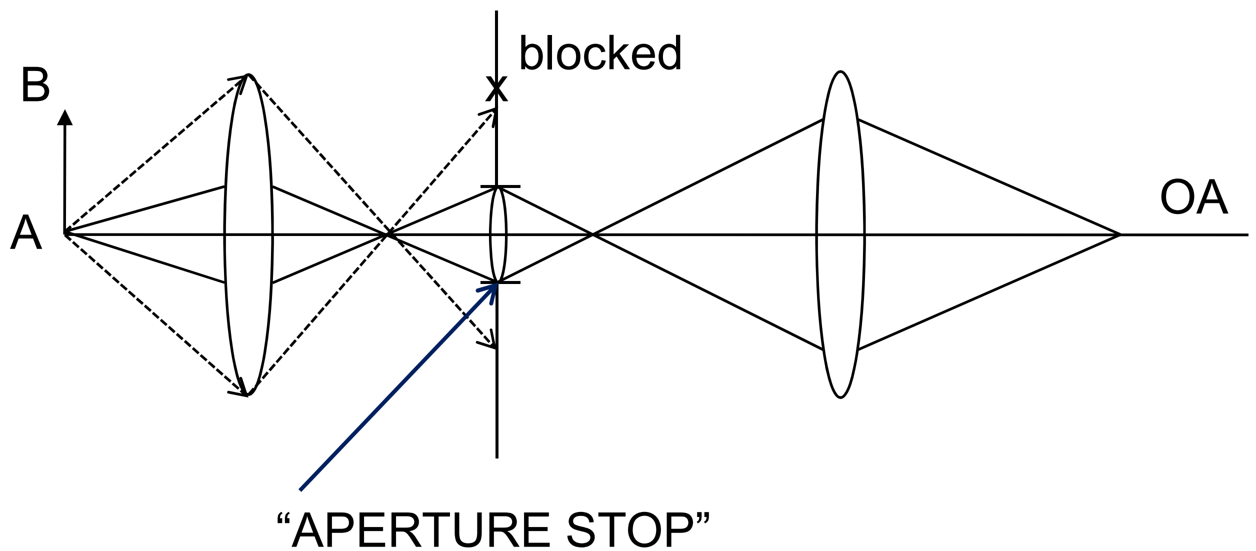
- Two main type of limitations:

  a) Solid angle Ω from the point on axis is limitted by the "ENTRANCE PUPIL"

  b) field of view (i.e extent of the object) limitted by "ENTRANCE WINDOW"

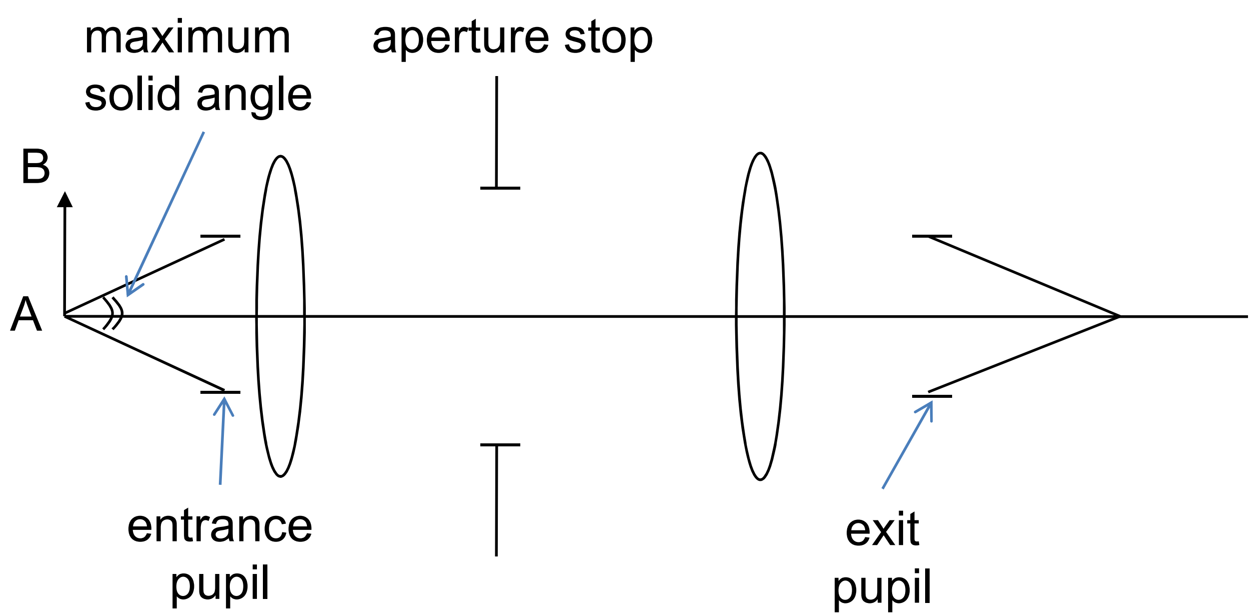
1. Pupils and Windows

a) Pupils



- Entrance Pupil ≡ image of aperture stop in the object space

1. Pupils and Windows



- <u>Exit Pupil</u> ≡ image of aperture stop in the <u>image</u> space

1. Pupils and Windows

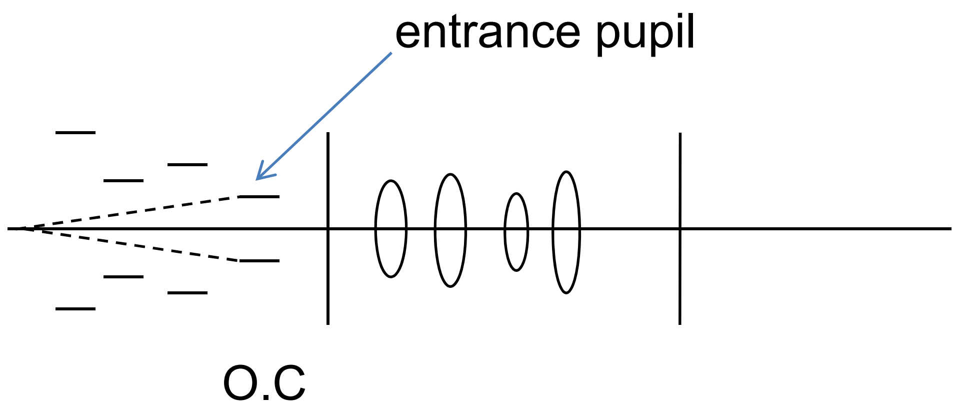
- Entrance Pupil:
  - Limits solid angle Ω
  - Thus, limits the amount of light, i.e brightness of image
  - Angle from object is proportional to spatial frequency

So:

  - Adjusting the pupil of our eyes, we adjust brigthness and resolution
  - High brightness = high resolution, but may induce saturation and aberations
  - Common issue in SLR photography

1. Pupils and Windows

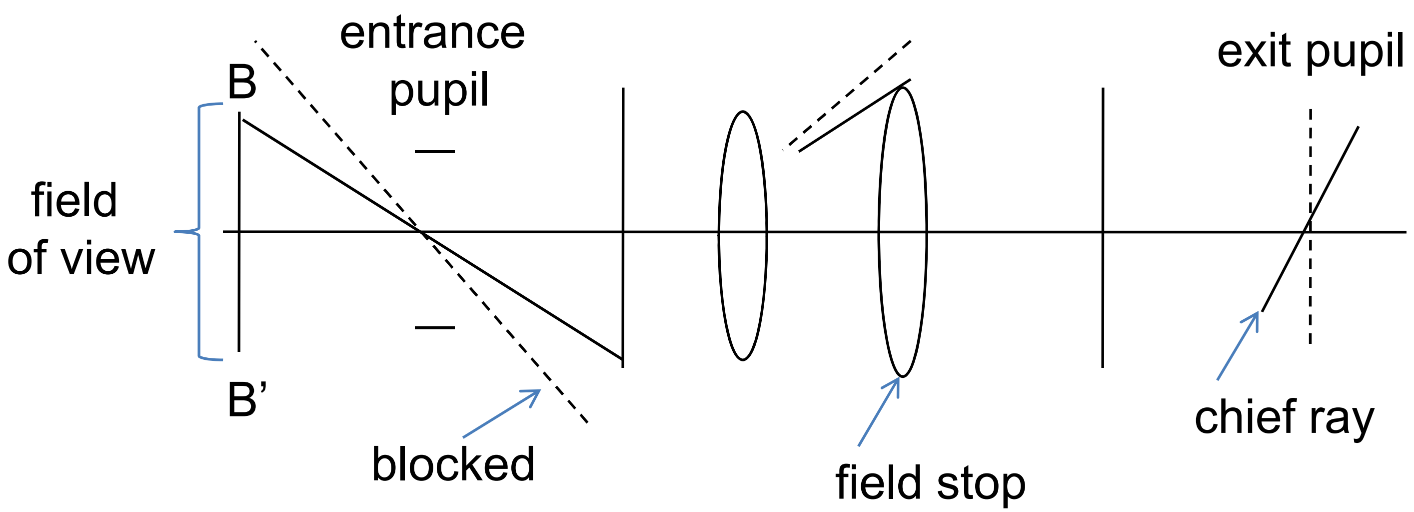
- Note: To find entrance pupil, image all optical elements in object plane and pick the one that subtend the smallest angle from object



1. Pupils and Windows

b) Windows

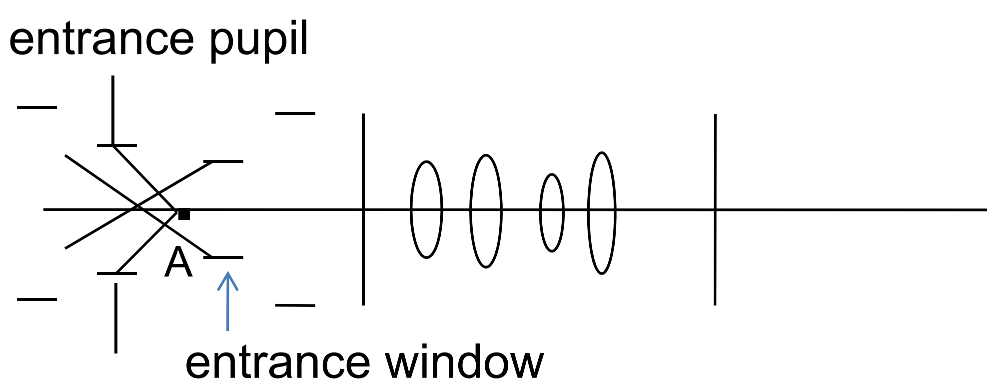
- Limi the solid angle made by rays at the center of the entrance pupil (chief rays) with OA



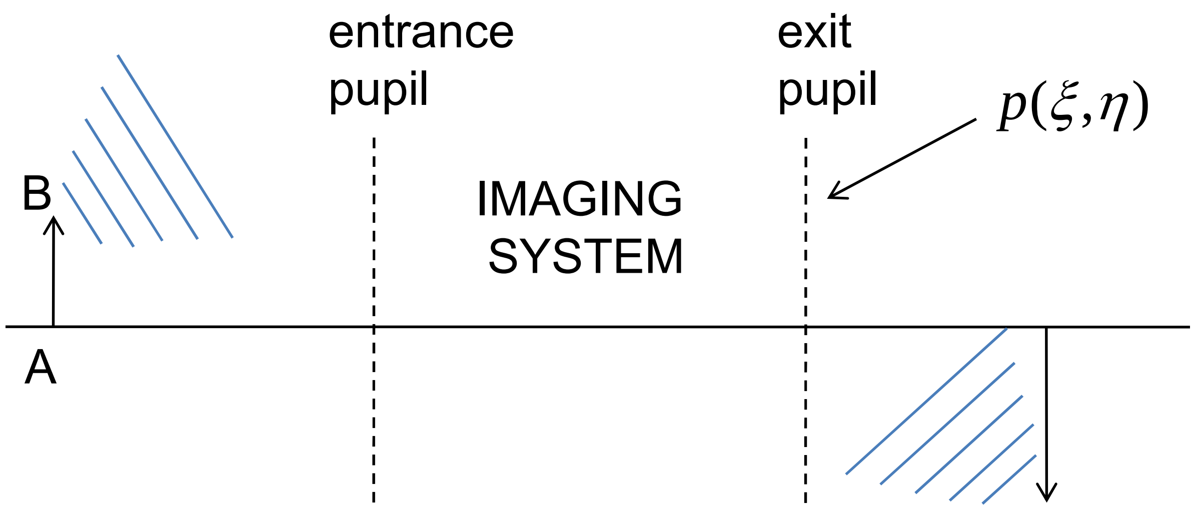
- Entrance Pupil = image of field stop in the object space
- Exit Pupil = image of the field stop in the image space

1. Pupils and Windows

- Limits the field of view
- Note:
  - Space and angle are Fourier related
  - It is always a compromise between field of view and solid angle (NA)
  - To find the entrance window: image all components in object space, and pick the one subtending the smallest angle from the center of the entrance pupil.



2. Frequency analysis of coherent imaging



- Recall that the image field equals the object field convolved with the impulse response of the system (Eq 4.4)

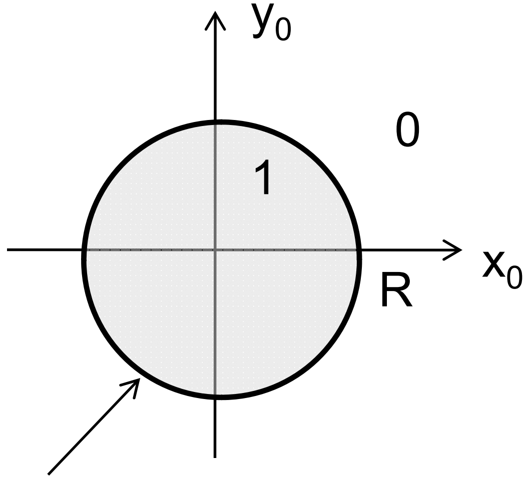
$$\overline{U}(x',y') = U(x,y) \circledast h(x,y) \qquad \text{1 a)}$$

$$h(x,y) = \Im[H(\xi,\eta)] = \int_{-\infty}^{\infty}\int H(\xi,\eta)e^{-i2\pi[x\xi+y\eta]}d\xi d\eta \qquad \text{1 b)}$$

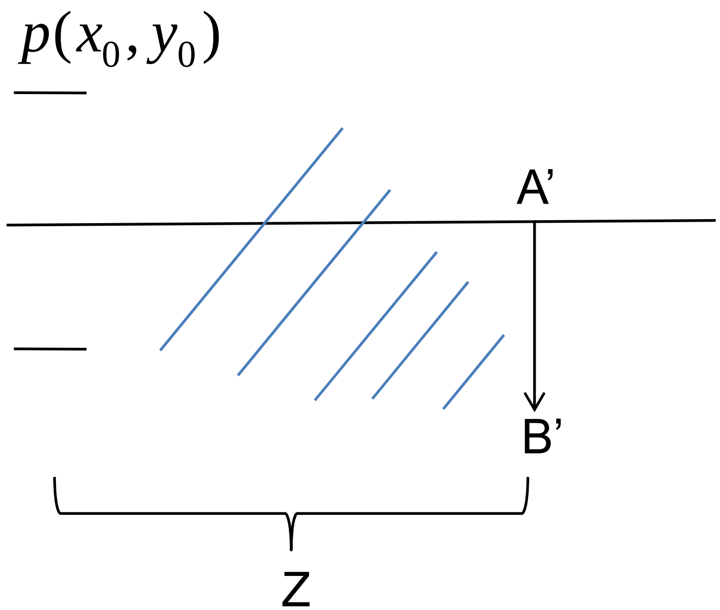
H = transfer function (coherent)

2. Frequency analysis of coherent imaging

- $p(x_0, y_0)$ = exit pupil
- Note: $\xi = \dfrac{x_0}{\lambda_z}; \eta = \dfrac{y_0}{\lambda_z} \Rightarrow H(\xi,\eta) = P(\lambda z\xi, \lambda z\eta)$



Example of a pupil



2. Frequency analysis of coherent imaging

- Most common pupil function:

$$p(x_0, y_0) = \begin{cases} 1, & {x_0}^2 + {y_0}^2 \leq R^2 \\ 0, & \text{otherwise} \end{cases} \qquad (2)$$

- So: Impulse response h is the Fraunhoffer diffraction pattern of P!
- This is a “diffraction-limitted” instrument, i.e. The best we can do in practice

3. Incoherent Imaging

- If the illuminating field is spatially incoherent, i.e:

$$\left\langle U(x_1,y_1;t)U^*(x_2,y_2;t)\right\rangle = I(x_1,y_1)\delta(x_1-x_2;y_1-y_2) \quad (3)$$

- The system becomes linear in intensities:

$$\bar{I}(x',y') = I(x,y) \textcircled{v} \, |h(x,y)|^2 \quad (4)$$

$$|h|^2 = h \cdot h^*$$

$$\Im[h \cdot h] \rightarrow \textcircled{v}$$

$$\Im[h \cdot h^*] \rightarrow \otimes$$

- The intensity impulse response: $|h|^2$!
- Incoherent transfer function:

$$H_1(\xi,\eta) = \Im[|h(x,y)|^2] = \Im[|h \cdot h^*] = H(\xi,\eta) \otimes H^*(\xi,\eta) \quad (5)$$

= autocorrelation of coherent transfer function

- Optical transfer function: $OTF = H_1(\xi,\eta) / H_1(0,0)$ (5)’

4. Effects of Aberrations

- So far, we assumed that points are imaged into points,

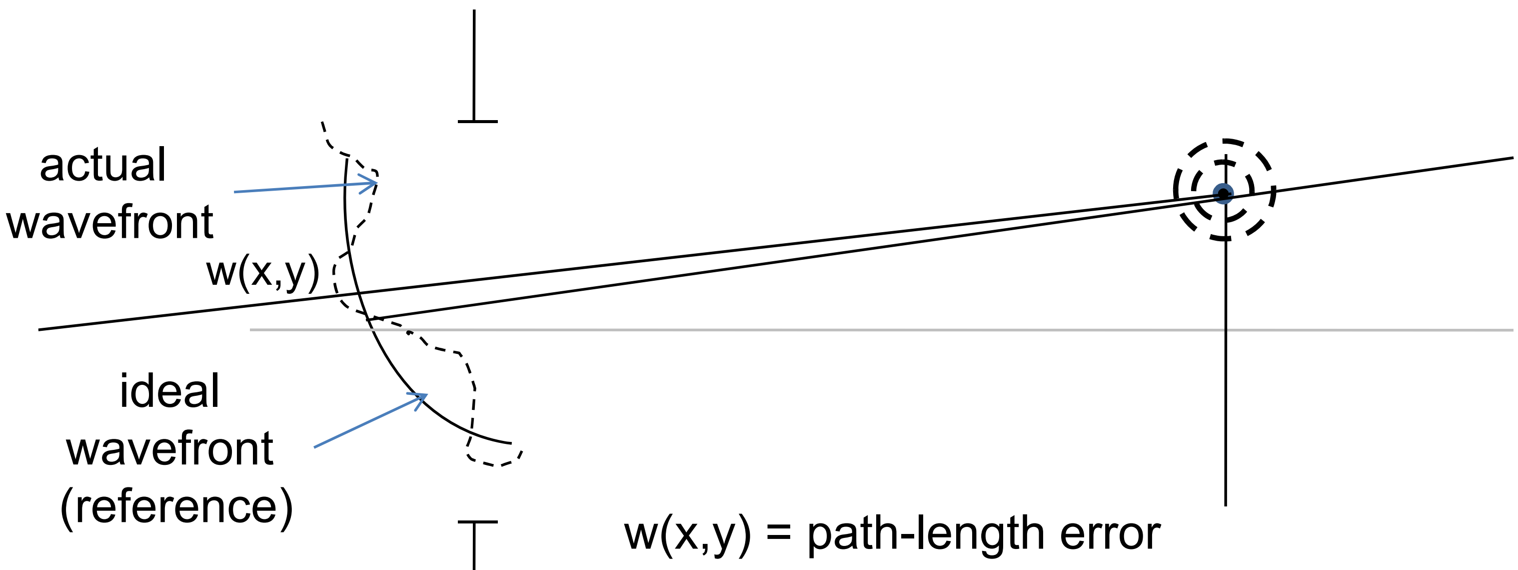
  i.e perfect imaging

  i.e diffraction-limitted system

  i.e point-source object generates a spherical wavefront at exit pupil

$$Sin[\theta] \nsim \theta$$

- In reality, the outgoing wavefront is nonspherical, i.e aberrated



4. Effects of Aberrations

All the effects of wavefront errors, i.e aberrations can be accounted for by generalizing the pupil functions:

$$P'(x,y) = P(x,y)e^{ikW(x,y)} \quad (6)$$

- Note:
  - W(x,y) can be positive or negative
  - W is the local difference between the actual wavefront and the reference one
- h and H follow from Eq (6):

  a) Coherent illumination:

$$\begin{cases} H'(\xi,\eta) = P'(\lambda z\xi, \lambda z\eta) = Pe^{ikW(x,y)} \\ h(x,y) = \Im[H(\xi,\eta)] \end{cases} \quad (7)$$

4. Effects of Aberrations

- W can have severe effects on the final image
- Exercise: Calculate h for $k \cdot W = a(\xi^2 + \eta^2)$

b) Incoherent Illumination:

- Recall Eq. 5:

$$H_1'(\xi,\eta) = H(\xi,\eta) \otimes H^*(\xi,\eta)$$
$$= P(\lambda z\xi, \lambda z\eta) e^{ikW(\lambda z\xi,\lambda z\eta)} \otimes [\quad]^* \quad (8)$$

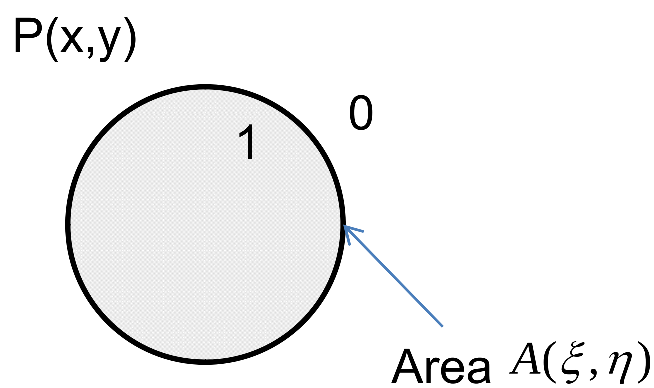
- Expressing the autocorrelation integral for a pupil function

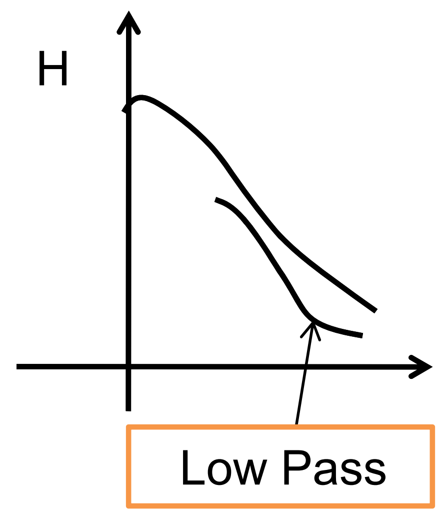
$$\begin{cases} P(x,y) = 1, inside \\ P(x,y) = 0, outside \end{cases}$$

$$\Rightarrow H_1'(\xi,\eta) = \iint_{A(\xi,\eta)} e^{ik\left[W\left(x+\frac{\Delta x}{2}, y+\frac{\Delta y}{2}\right) - W\left(x-\frac{\Delta x}{2}, y-\frac{\Delta y}{2}\right)\right]} dxdy \quad (9)$$

4. Effects of Aberrations

$$\begin{cases} \xi = \dfrac{x}{\lambda z} \\ \eta = \dfrac{y}{\lambda z} \end{cases}$$





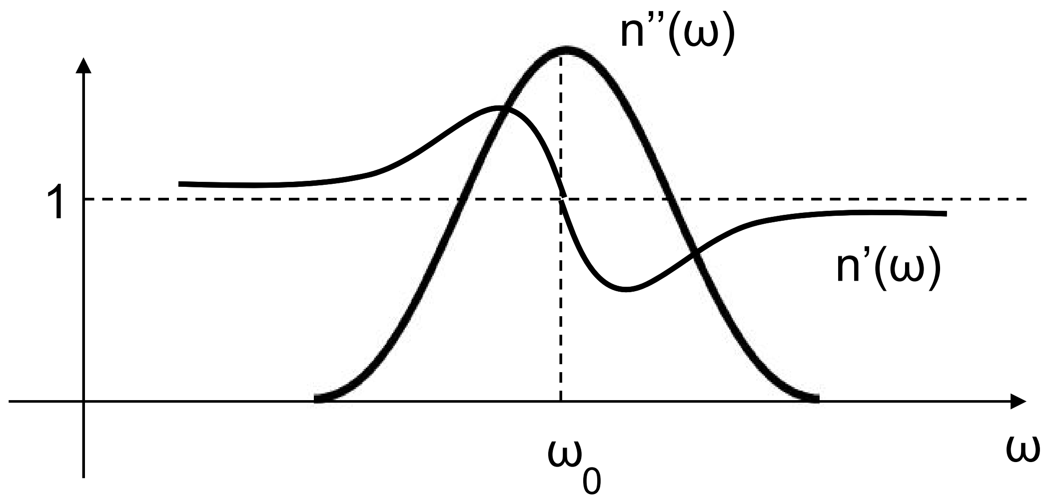
- Note: $\left|H_1'(\xi,\eta)\right|^2 \leq \left|H_1(\xi,\eta)\right|^2$ (10)

→Aberrations can only reduce high freq of power spectrum

→Note that k=2π/λ ➔ wavelength dependence can have an efect, too! Important in broad spectrum imaging (e.g. white light).

5. Chromatic Aberrations

- Recall the dispersion curve from chapter 1.8 (Lorentz model):

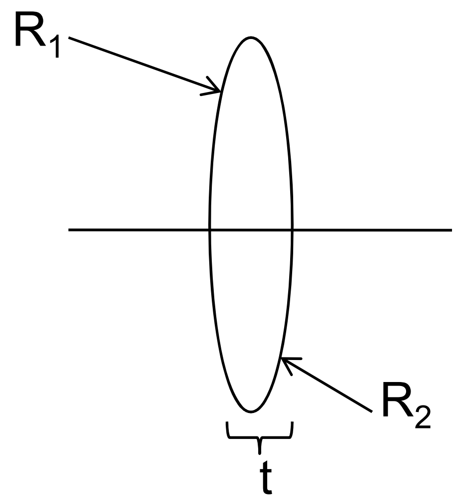


- n' – refractive index
- n'' – absorption

- We assume n'' → 0, i.e our optical components are higly transparent

5. Chromatic Aberrations

- Consider a lens made of glass with n'(ω)



$R_1 > 0$

$R_2 < 0$

- <u>Convergence</u> $C = C_1 + C_2 - C_1 C_2 \frac{t}{n}$

$$= \frac{n-1}{R_1} + \frac{1+n}{R_2} - \frac{(n-1)(1-n)t}{nR_1R_2} \qquad (11)$$

5. Chromatic Aberrations

- Thin Lens: t = 0

$$\Rightarrow C = \frac{1}{f}$$

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \qquad (12)$$

- If $|R_1| = |R_2|$

$$\Rightarrow \boxed{\frac{1}{f} = \frac{2(n-1)}{R}} \qquad (13)$$

5. Chromatic Aberrations

- The dispersion effect is very clear:

$$f(\omega) = \frac{R}{2[n(\omega)-1]} \Rightarrow \text{focal distance varies with color!} \quad (14)$$

- Differentiate Eq. 14:

$$d\left[\frac{1}{f(\omega)}\right] = d\left[\frac{2(n-1)}{R}\right] \quad (15)$$

$$-\frac{1}{f^2}\frac{df}{d\omega} = \frac{2}{R}\frac{dn}{d\omega} \quad (16)$$

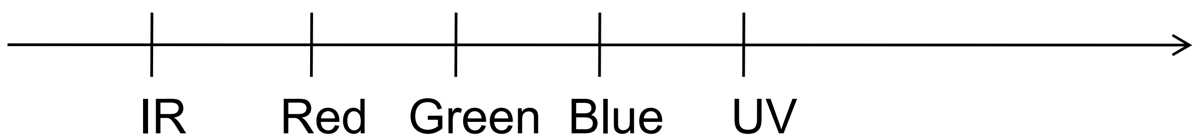
$$\boxed{\frac{df}{d\omega} = -\frac{2f^2}{R}\frac{dn}{d\omega}} \quad (17)$$

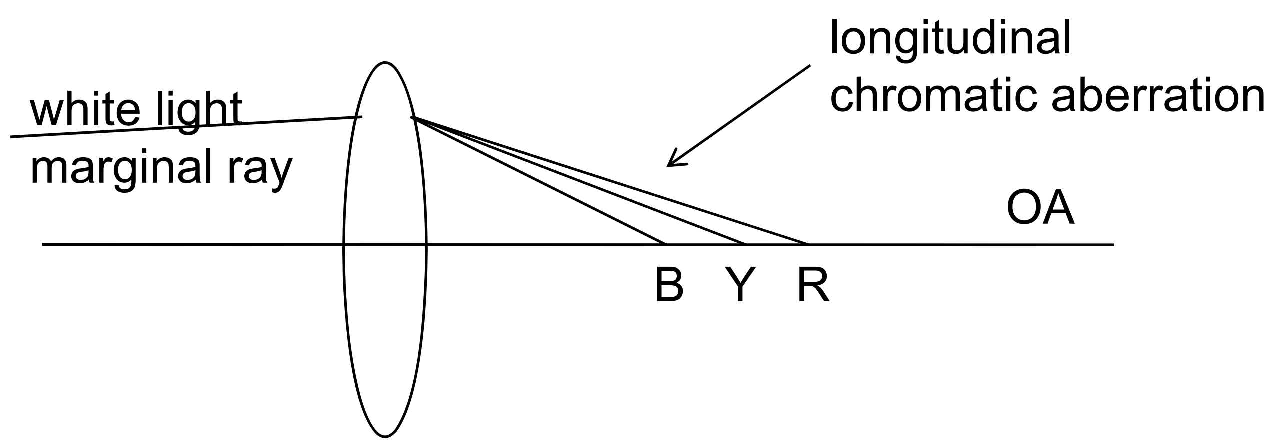
5. Chromatic Aberrations

$$\frac{df}{d\omega} = -\frac{2f^2}{R}\frac{dn}{d\omega}$$

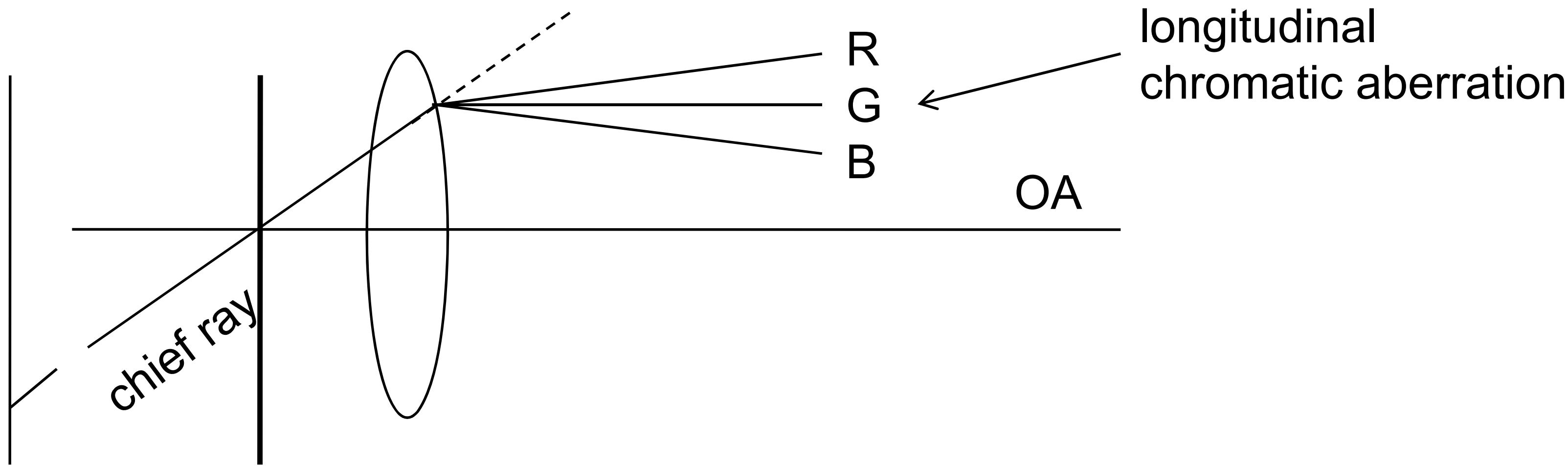
$$\frac{dn}{d\omega} > 0 \;\Rightarrow\; \text{normal dispersion}$$

- What is the negative sign telling?





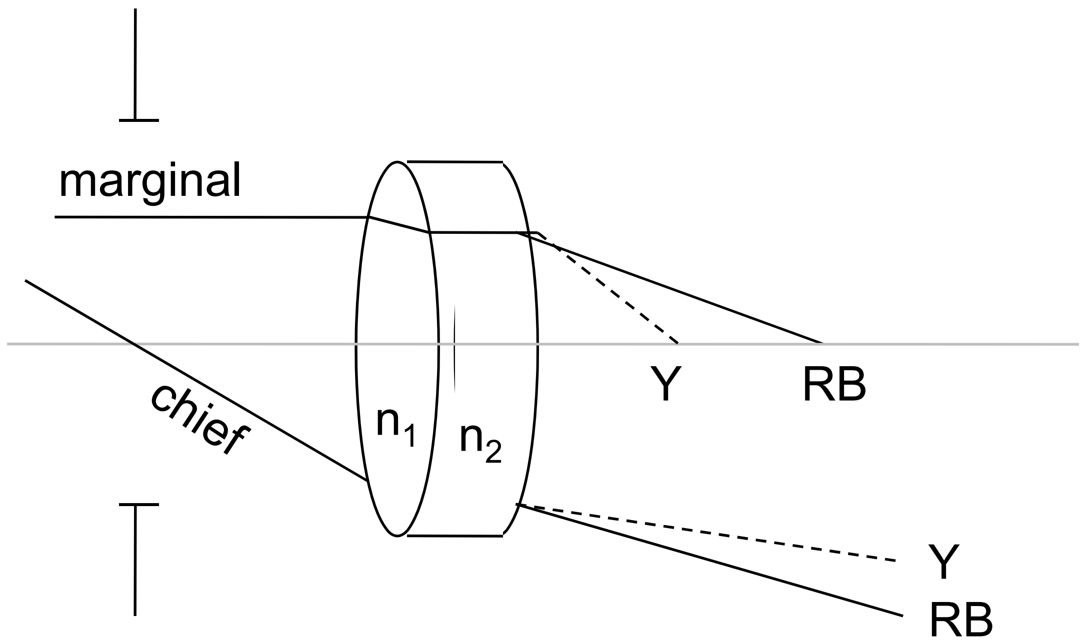
5. Chromatic Aberrations



- Most common correction: the achromatic doublet

→ Sandwich of two lenses, one positive, one negative, which corrects the chromatic aberration at 2 colors (R&B)

5. Chromatic Aberrations



- For thin, closely packed lenses:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad (18)$$

5. Chromatic Aberrations

- For thin, closely packed lenses:

$$\frac{1}{f(\omega_1)} = \frac{1}{f(\omega_2)} \Rightarrow \frac{2\left[n_1(\omega_1)-1\right]}{R_1} + \frac{2\left[n_2(\omega_1)-1\right]}{R_2} =$$

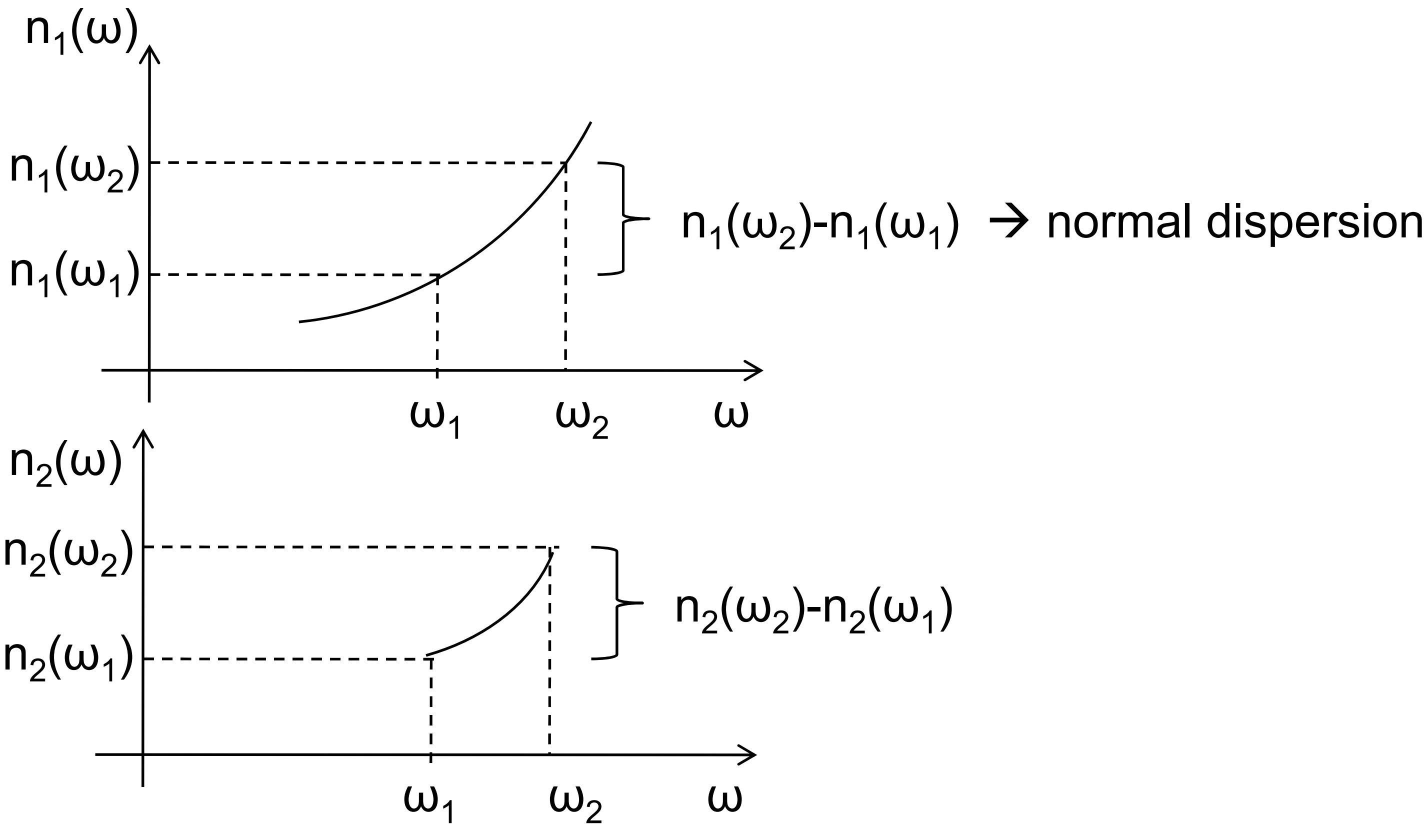
$$= \frac{2\left[n_1(\omega_2)-1\right]}{R_1} + \frac{2\left[n_2(\omega_2)-1\right]}{R_2} \qquad (19)$$

$$\Rightarrow \frac{n_1(\omega_1)-n_1(\omega_2)}{R_1} = -\frac{n_2(\omega_1)-n_2(\omega_2)}{R_2} \qquad (20)$$

R_2 = negative

$$\Rightarrow \boxed{\frac{n_2(\omega_1)-n_2(\omega_2)}{n_1(\omega_1)-n_1(\omega_2)} = \frac{R_2}{R_1}}$$

5. Chromatic Aberrations



- This corrects for 2 colors

! A microscope objective may contain more than 8 lenses

→ correction to multiple colors

6. Geometric (monochromatic) aberrations

- Sinϴ = ϴ → paraxial approximation

$-\frac{\theta^3}{3!} \rightarrow$	Third order aberrations (Seidel) = primary aberrations

$-\frac{\theta^5}{5!} \rightarrow$ Higher order

- 3rd order aberrations:

a)Spherical aberration

b) coma

c) astigmatism

d) field cuvarture

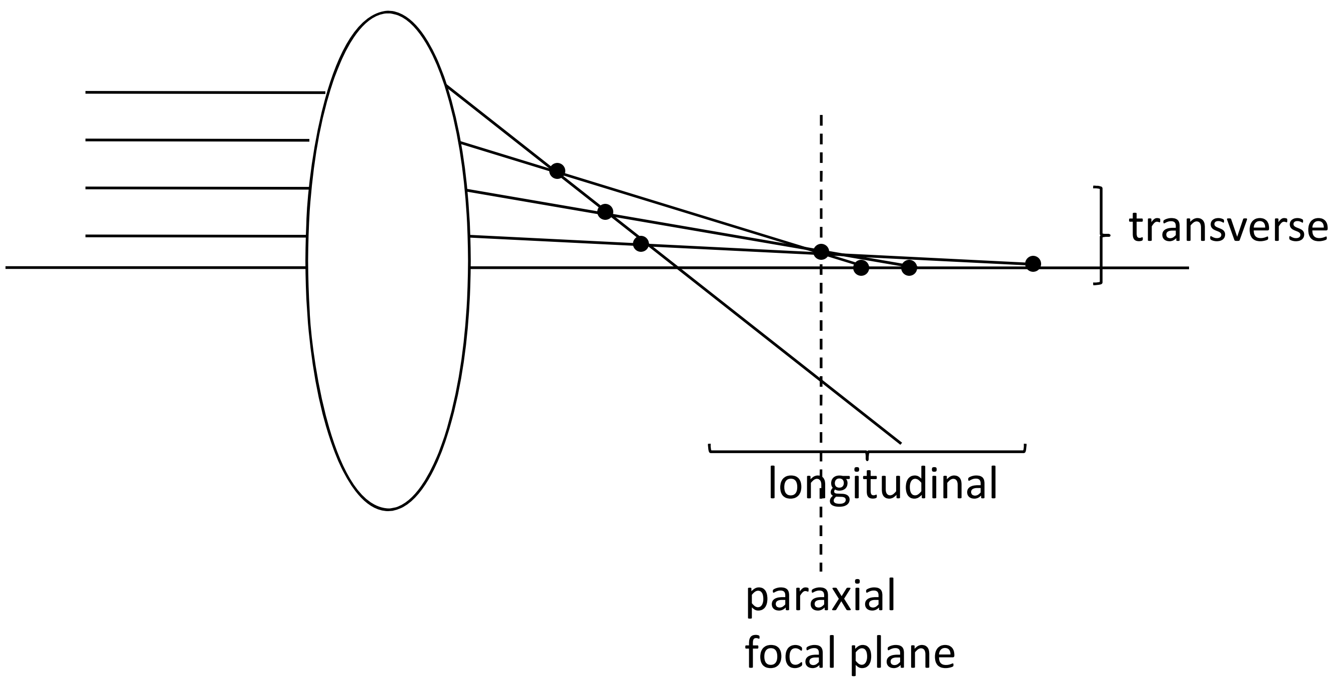
e) distortion

Let's see what each means

6. Geometric (monochromatic) aberrations

a) Spherical aberration

- Focal length depends on aperture



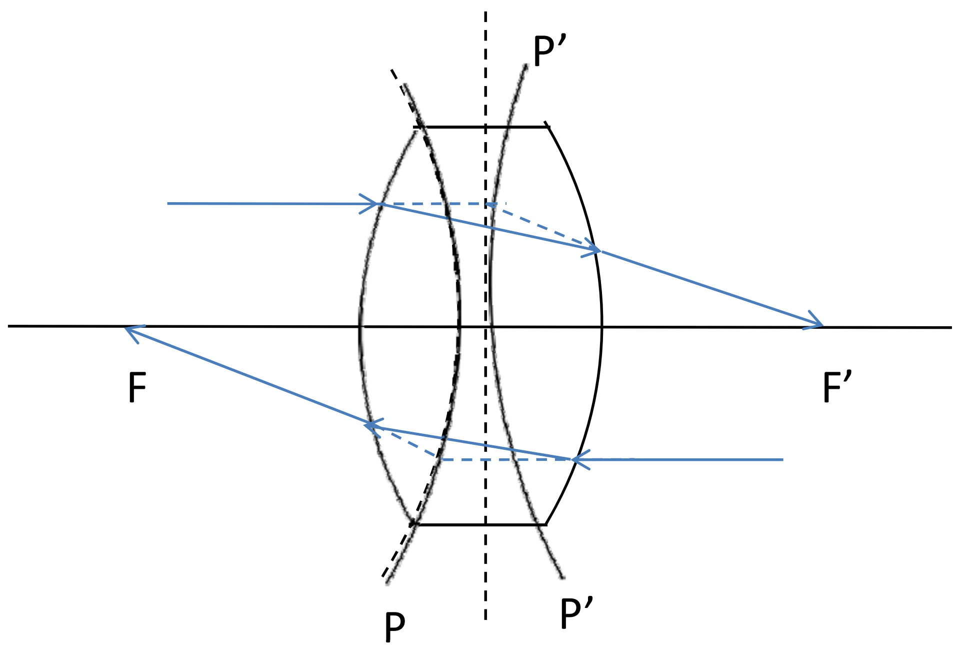
6. Geometric (monochromatic) aberrations

- To avoid SA in practice:
  - Stay close to optical axis
    (small aperture)
  - Polish off edge of lens
    (as in telescopes, microscope objectives)

6. Geometric (monochromatic) aberrations

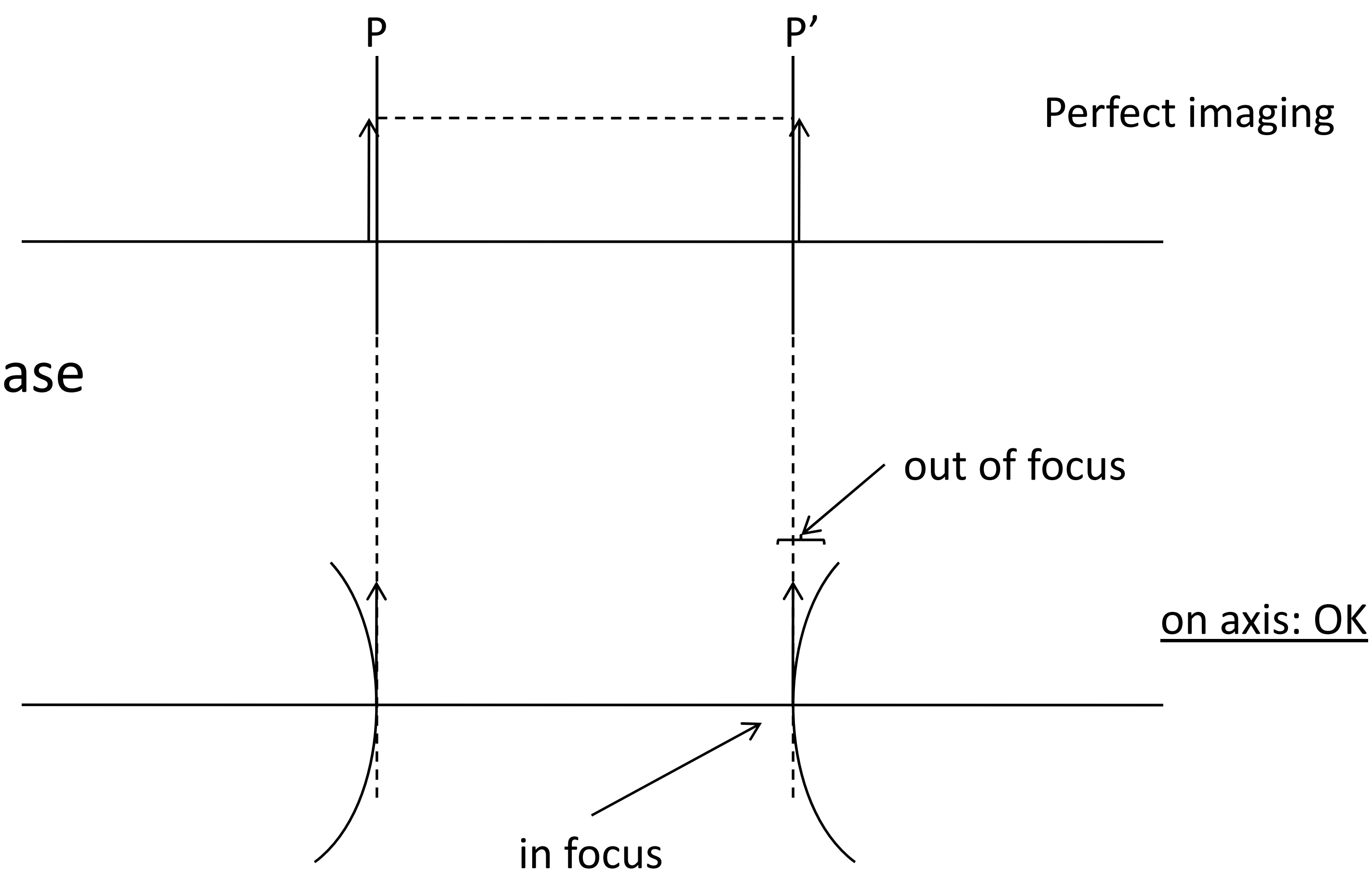
b) Coma

- Occurs because principle planes (i.e. locus of conjugate points image with magnification 1) "curve" at large angles

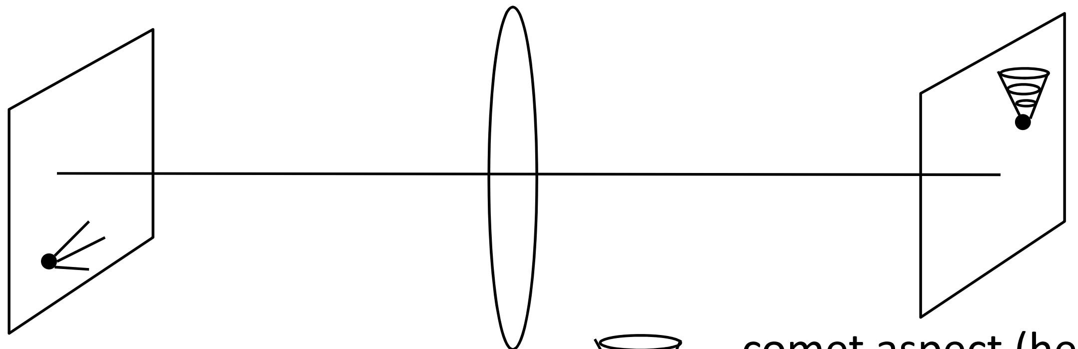


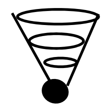
6. Geometric (monochromatic) aberrations

- Paraxial Approximation:
- Real case



6. Geometric (monochromatic) aberrations

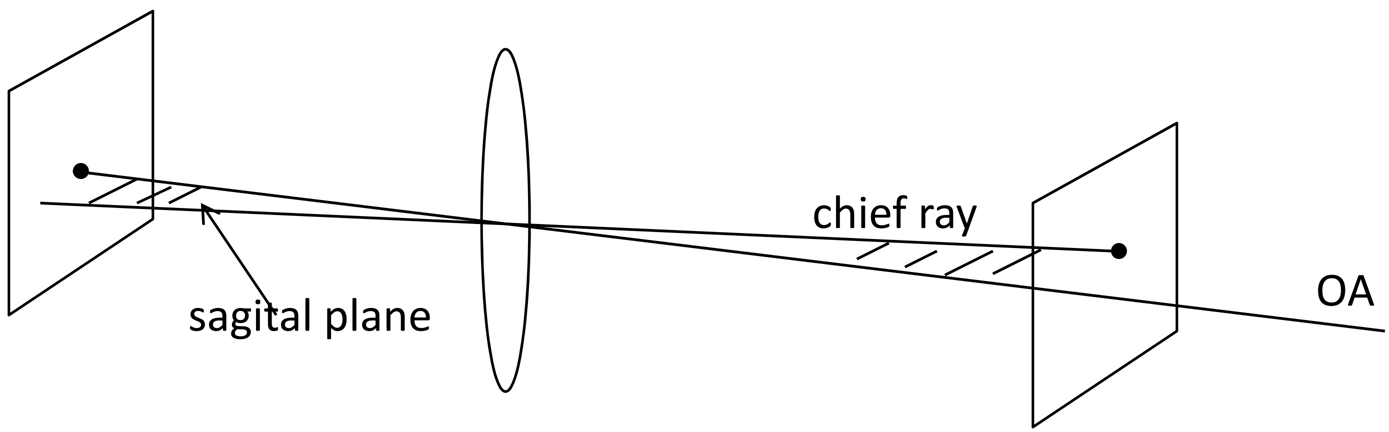




- comet aspect (hence the name)
- use stops to improve image quantity

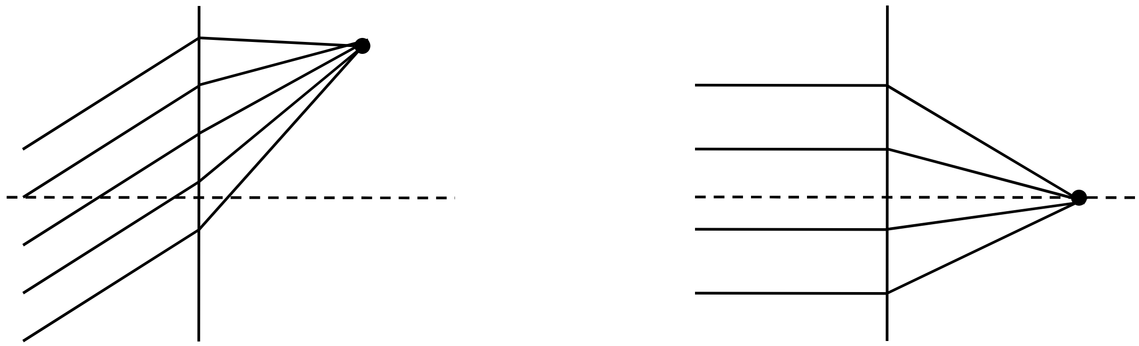
c) Astigmatism

- Different focus on perpendicular planes
  - On axis, no difference between the planes



6. Geometric (monochromatic) aberrations

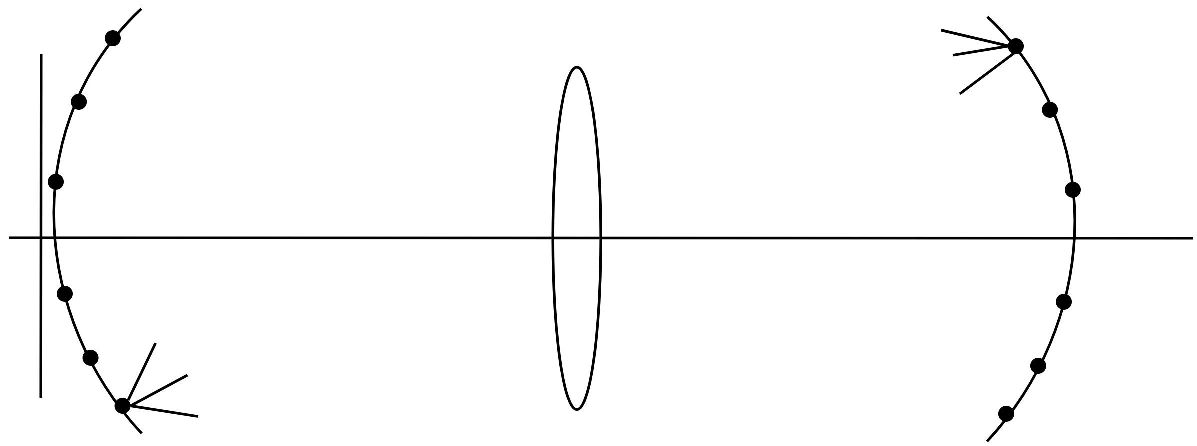
- Meridional plane is perpendicular to sagital



meridional plane sagital plane

6. Geometric (monochromatic) aberrations

d) Field curvature

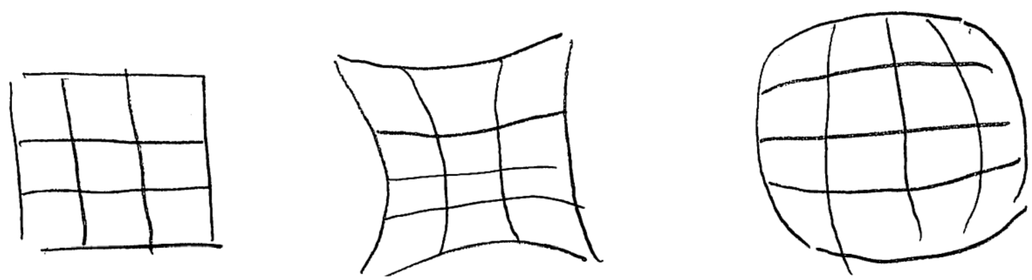


- With astigmatism: 2 paraboloidal surfaces

6. Geometric (monochromatic) aberrations

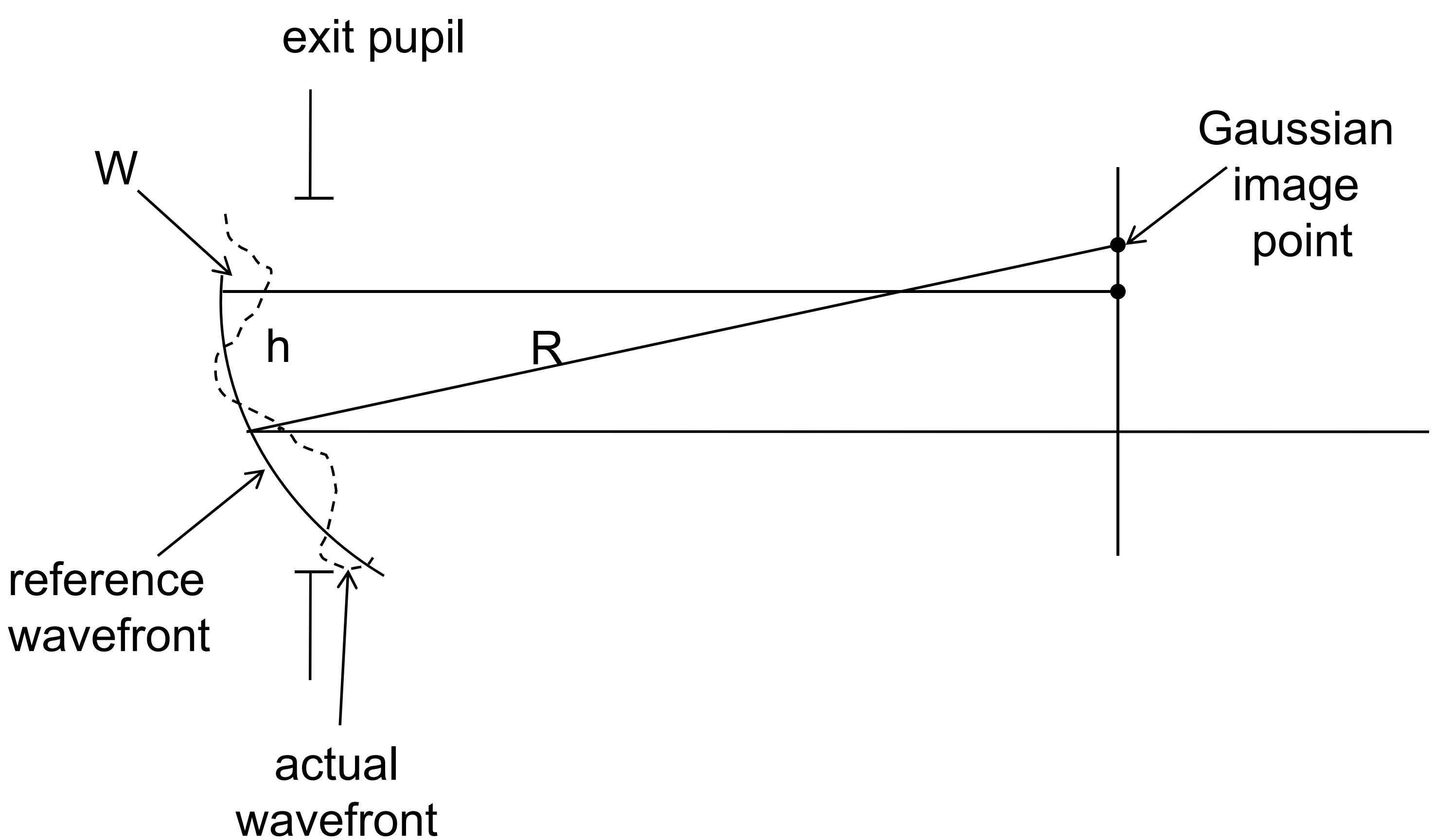
e) Distortion

- Transverse magnification is a function of height



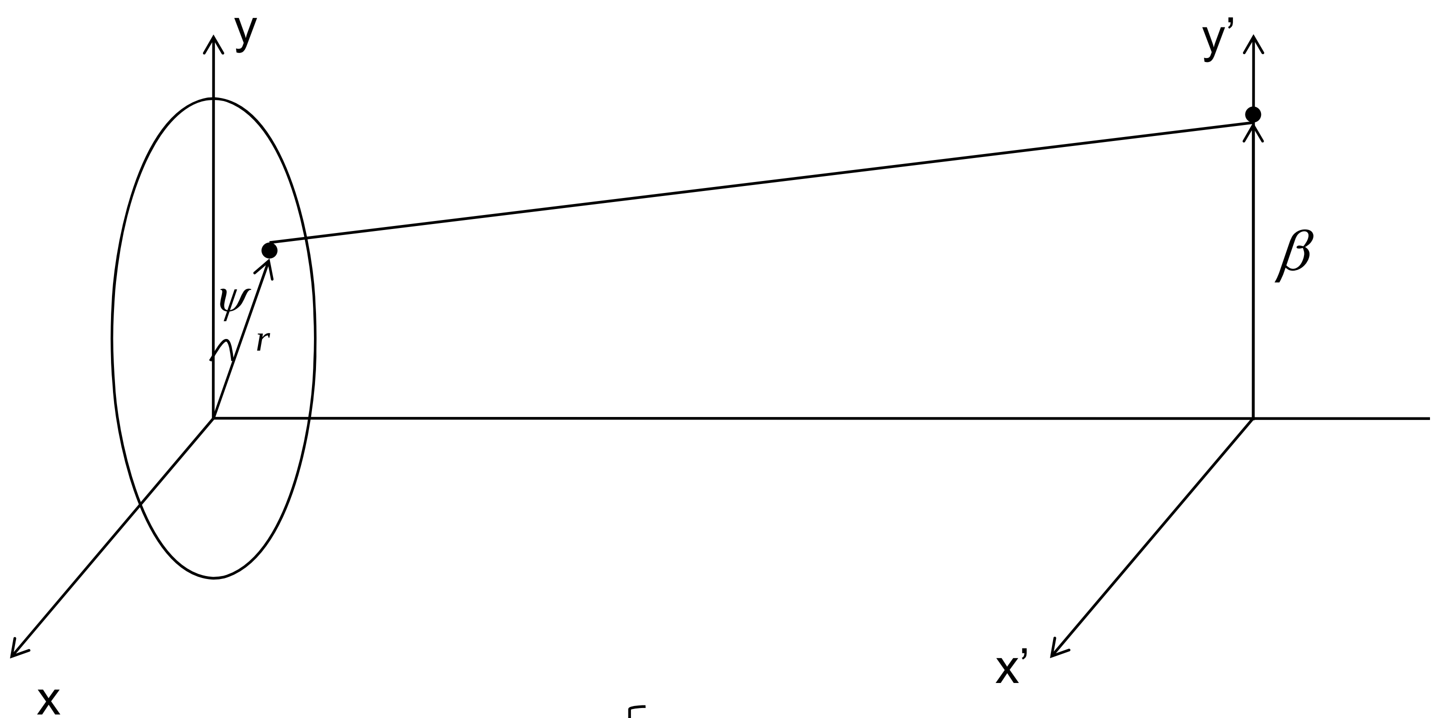
a) undistorted b) cushion (positive) c) barrel (negative)

7. Summary of Geometric Aberrations



W = wavefront aberration

7. Summary of Geometric Aberrations



$$W = f(\beta, r, \psi)$$

$\beta =$ normalized field height
$r =$ normalized pupil height
$\psi =$ azimuth angle

7. Summary of Geometric Aberrations

- Symmetries:

$$W(\beta, r, \psi) = W(-\beta, -r, -\psi)$$

→W depends only on $\beta^2, r^2, \beta r \cos\psi$

If $\beta = 0$ (on axis) → W independent on ψ

→ W depends on $\beta r \cos\psi$

→ $$\boxed{W = W(\beta^2, r^2, \beta r \cos\psi)}$$

7. Summary of Geometric Aberrations

$$\Rightarrow W(\beta, r, \psi) = W_{000} +$$

$$+\cancel{W_{200}}\beta^2 + \underbrace{W_{020} r^2}_{\text{defocus}} + \underbrace{W_{111}\beta r \cos\psi}_{\text{tilt}} +$$

$$+\cancel{W_{400}}\beta^4 + \underbrace{W_{040} r^4}_{\text{SA}} + \underbrace{W_{131}\beta r^3 \cos\psi}_{\text{coma}} +$$

$$+\underbrace{W_{222}\beta^2 r^2 \cos^2\psi}_{\text{astigmatism}} + \underbrace{W_{220}\beta^2 r^2}_{\text{field curvature}} +$$

$$+\underbrace{W_{211}\beta^2 r \cos\psi}_{\text{distortion}} + \ldots$$