**Objective**

(1) To understand the superposition of fields leading to interference patterns.

(2) To understand the concept of coherence as applied to interference and the relationship between the temporal and spectral properties of light.

(3) To measure the power spectral content of a He-Ne laser.

**Background**

- As discussed in class, when two monochromatic waves are superposed the resulting intensity pattern (known as the interference pattern) is the magnitude squared of the sum of the two complex amplitudes.

\[ I = |U_1 + U_2|^2 = |U_1|^2 + |U_2|^2 + 2|U_1||U_2|\cos(\phi_1 - \phi_2) \]  

where \( U_1 = |U_1|e^{j\phi_1} \) and \( U_2 = |U_2|e^{j\phi_2} \) are the complex amplitudes of the two waves. We’ll be exploring this interference pattern in this lab by setting up a Michelson interferometer. Refer to Section 2.5 in Saleh and Teich further reading.

- So far, we have been assuming that the laser is purely monochromatic. This week we depart from the assumption that the laser’s power spectral density \( S(v) \) is a delta function. Instead, we realistically consider it to be a function with a finite width, such as a Gaussian or Lorentzian. This has profound implications on the propagation of optical waves.

The temporal coherence function \( G(\tau) \) is a measure of the ability of a wave \( U_1 \) to interfere with a time-shifted version of itself \( U_2 = U_1(t + \tau) \). A flat coherence function implies that \( U_1 \) and \( U_2 \) will form an interference pattern even for long time delays \( \tau \). \( G(\tau) \) forms a Fourier transform pair with \( S(v) \). For non-monochromatic light the interference equation (1) becomes

\[ I = |U_1 + U_2|^2 = |U_1|^2 + |U_2|^2 + 2|U_1||U_2|g_{12}(\tau)\cos(\phi_1 - \phi_2) \]  

where \( g_{12}(\tau) = \frac{\delta(\tau)}{|U_1||U_2|} = \frac{<U_1^*U_2>}{|U_1||U_2|} \).

We will study the effect of this finite spectral width on the ability to form a stationary interference pattern. Refer to Section 11.2 in Saleh and Teich for further reading.

**PRELAB QUESTIONS**

1) Consider the geometry shown in the figure below. Sketch the shape of interference pattern you would expect to see on the screen due to interference between light from monochromatic point sources \( s_1 \) and \( s_2 \) for the cases (a) \( d = 0 \) and (b) \( d \neq 0 \). Assume the screen is far away from the sources such that \( R_1 >> d \) and \( R_2 >> d \). (4 points)
2) All photodetectors detect intensity, which is the time average of the magnitude squared of the field. (Refer to your lecture notes or text.) Let the incident field on a photodetector be written as

\[ u(t) = e^{j\omega_1 t} + e^{j\omega_2 t} \]  

(Sum of harmonics.)

where \( \omega_1 - \omega_2 \ll \omega_1 \) or \( \omega_2 \).

i) Determine the frequency of the intensity on the photodetector. (Hint: determine \( u(t)u^*(t) \))

ii) If \( \omega_1 \) and \( \omega_2 \) are adjacent modes of a multimode laser, determine the beat frequency as a function of the size of the cavity. (Hint: the mode spacing (Hz) on a laser is \( c/2d \) where \( d \) is the cavity length.)

(4 points)

3) Many He-Ne lasers have a power spectral density \( S(v) \) that can be modeled (simply) as

\[ S(v) = \sum_{m=-\infty}^{\infty} \gamma(v) \delta(v-mv_c) \]

In this equation, \( v_c = c/(2d) \) is the free spectral range (i.e. mode spacing) of the cavity. The function \( \gamma(v) \) is called the lineshape and is modeled for the He-Ne laser as

\[ \gamma(v) = \frac{\Delta v/2\pi}{(v-v_o)^2 + (\Delta v)^2} \]

where \( v_o \) is the operating frequency of the laser and \( \Delta v \) measures the frequency spread. (The function is called a Lorentzian.)

i) Using the shift property of Fourier transforms and an appropriate Table from the Appendix, determine the Fourier Transform of \( \gamma(v) \). (Hint: this is two exponentials back-to-back at the origin.)

ii) Using the results of part i), determine the coherence function \( G(\tau) \). (Hint: \( S \) and \( G \) are Fourier transform pairs.)

(4 points)
PART I  MICHELSON INTERFEROMETER: MEASURING SOURCE WAVELENGTH

Discussion
Since the Michelson interferometer is sensitive to optical path-length changes on the order of less than the wavelength of light, we will be using it in this part to determine the wavelength of a HeNe laser as well as of Sodium lines within a Sodium-Tungsten lamp.

Alignment
We will be making use of the Lambda Scientific Systems, Inc. precision interferometer kit (LEOI - 22) for this part of the lab. The figure below shows an excerpt of the instrument manual which shows a schematic of the interferometer.

Before beginning, make sure the HeNe laser source (part # 3) and screen (part # 16) are installed as shown. Perform the following steps to align the interferometer
A. With the beam expander removed, adjust the laser height and tilt settings until the laser spot on both the beam splitter (part # 14) and the moveable mirror (part # 11) is centered on both components.
B. Place a business card in front of the moveable mirror (part # 11). You will observe the light from fixed mirror (part # 8) hitting the screen (part # 16). Adjust the laser until the brightest spot hits the center of the screen (part # 16).
C. Now remove the business card from in front of the moveable mirror (part # 11). You will see light from both mirrors hitting the screen. Adjust the knobs on the moveable mirror (part # 11) until the two brightest spots overlap
D. Now install the beam-expander (part # 4) in SOCKET 2 (part # 17) to observe the interference fringes

Experiment
We will now use this interferometer to measure the wavelength of two different light source, beginning with the HeNe laser already installed.

1) After obtaining the interference pattern, note down the reading \( d_0 \) of the fine micrometer (part # 10). Turn this micrometer and count the number of fringes \( N \) that collapse/expand into/out of the center of the interference pattern. Stop at \( N = 50 \) and note down the final reading \( d_{\text{final}} \) of the fine
micrometer (part # 10). Use \( d_0 \), \( d_{\text{final}} \) and \( N \) to determine the wavelength \( \lambda \) of HeNe laser

\[ \text{(4 points)} \]

2) Replace the HeNe laser with the Sodium-Tungsten lamp and repeat to obtain the wavelengths of the Sodium D lines. Use the mirror end of the screen to observe fringes in this part of the experiment.

\[ \text{(4 points)} \]

PART II  
MODE SPACING OF A HeNe USING MODE BEATING ON A PHOTODETECTOR

Discussion  
Most He-Ne lasers ‘lase’ in several modes separated by the free spectral range (FSR) of the Fabry-Perot cavity. These cavity modes (frequencies) are all present in the laser beam and will beat at a receiver. This induces a sinusoidally-varying current, whose frequency spectrum can be displayed on an electrical spectrum analyzer.

Experiment  
Couple the laser light into the fiber using the lens and fiber mount, as shown in the figure. The fiber should be disconnected from the receiver, and you should hold the fiber up to a white piece of paper for alignment. Once you have successfully coupled sufficient light into the fiber, connect the fiber output to the receiver. Send the output of the receiver through the electrical amp (built into the receiver) and then into the electrical spectrum analyzer. Find the RF frequency resulting from the beating of two adjacent cavity modes. What is the laser FSR? Center this signal with 3MHz span. Explain the spectrum you see.

\[ \text{(4 points)} \]

PART III  
\( S(\nu) \) FOR AN HE-NE LASER USING FABRY PEROT INTERFEROMETER
**Discussion**

From question (3) (ii) of Pre-lab we note that a very long length $d$ is required to determine the frequency structure of a He-Ne laser using a grating. Thus, in order to see the structure in more detail we will use a Fabry-Perot interferometer.

**Alignment**

Set the experiment up as shown in the figure above. The TA will help you align the laser into the interferometer. If you have not done this before, it is easier to show than to explain. This will take some time. If it was just turned on, the HeNe will require at least 10 minutes to gain thermal stability. Until stable, you will observe the spectrum drifting around quite a bit. This is not a trigger issue on the scope.

**Experiment**

1) Once the interferometer is aligned, use the controller to produce 1 free spectral range over the entire scope trace. Calibrate the Fabry-Perot (MHz/ms) knowing that the specification for the free spectral range (fsr) is 1.5 GHz. The TA will help you if you are confused about this.

2) Using this calibration, determine the mode spacing for the laser. (4 points)

3) Save a copy of the data if this is available. (1 point)

4) Turn the laser off for a few minutes and then turn it back on. Set the scope to infinite persistence (this is the ‘Autostore’ button on HP scopes.) and explain what you see on the trace. As the lines move, use the cursors to determine the Full Width Half Max (FWHM) of envelope that the modes sweep out. This envelope is the gain profile of the He-Ne. (4 points)

**PART IV**

**G(τ) FOR A HE-NE LASER**

**Discussion**

Coherence may be considered the ability of two beams of light to interfere. When two beams interfere, the intensity of the sum is greater (or less) than the individual intensities. The degree of coherence can be determined by the contrast or visibility of the fringes.

**Experiment**

1) The basic set-up shown in the figure will be provided. Set the distance from M1 and M2 to the beamsplitter to be equal. Using only the tilt adjustments on M1, try to produce as few interference fringes as possible. (i.e one fringe over the screen) Sketch the intensity pattern you achieve. Explain why it is not uniform. (2 points)

2) Adjust M2 until the fringes are vertical and linear. Move M1 in increments of 5 cm and take 8 readings until you reach 40 cm. Measure the following quantities using the cursors on the scope.
i) The relative intensity of arm 1 (block arm 2) ($I_1$).

ii) The relative intensity of arm 2 (block arm 1) ($I_2$).

iii) The maximum value of the intensity of the sum ($I_{\text{max}}$, from the fringes)

iv) The minimum value of the intensity of the sum ($I_{\text{min}}$, from the fringes)

Be sure that the measurements are made at the same place on the trace (Why?) These values will allow you to calculate the relative visibility function and thus the coherence function $G(\tau)$. Note: you only have to perform measurements i) and ii) once, while iii) and iv) have to be done for each mirror position. (4 points)

3) We will provide you with a complete data set for $I_{\text{max}}$, $I_{\text{min}}$, $I_1$, and $I_2$ for 40 points via the class Web site. (It will take you too long to do this yourself.) From this data, plot out the magnitude of the normalized coherence function $|g_{12}|$. (4 points)

QUESTIONS FOR WRITEUP

1) What was the purpose of the compensator (part #13) in Part I of this lab? (2 points)

2) In Part III, why did the spectrum flop around when you turned the laser back on? (2 points)

3) Compare the free spectral range of the He-Ne laser measured from the coherence function, the ESA, and from the Fabry-Perot. Try to explain any differences between the three measurements. (4 points)

4) Following a procedure similar to that in Lab 1, calculate magnitude of the Fourier transform of $G(\tau)$ from Part IV, which is $S(\nu)$, and compare with the spectrum measured in Part III. Be sure to correctly get the scaling between $\Delta t$, $\Delta \nu$, and $nn$ where $nn$ is the total number of points in the trace, to be sure that the relative spectrum is correct. (4 points)