11.1-4) \( \Delta \lambda = \Delta V \cdot \frac{\lambda^4}{c} = 16.3 \text{ mm} \). \( T_c = \frac{0.32}{\Delta V} = 3.2 \times 10^{-14} \text{ s} \) for Lorentzian spectra. \( l_c = c T_c = 9.6 \mu m \).

Note, for Lorentzian spectra, its autocorrelation function is exponential delay \( \Rightarrow \rho(t) = e^{i2\pi \nu_0 t} - e^{i2\pi \nu_1 t} \)

\( |g(t_{\text{max}})| = e^{-\pi \nu_0 t_{\text{max}}} = \frac{1}{2} \Rightarrow t_{\text{max}} = \frac{\nu_2}{\pi \nu_0} = 22 \text{ fs} \)

11.1-8)

a) Narrowband source

\[ \Delta \nu = \nu, -\nu = \frac{c}{\lambda_{\text{avg}} + \frac{1}{2} \Delta \lambda} - \frac{c}{\lambda_{\text{avg}} - \frac{1}{2} \Delta \lambda} = \frac{c \Delta \lambda}{\lambda_{\text{avg}}^2} \]

\[ l_c = \frac{c}{\Delta \nu} = \frac{\lambda^4}{\Delta \lambda} \]

b) Broadband source

\[ \frac{c \Delta \lambda}{\lambda_{\text{avg}} - \frac{1}{2} \Delta \lambda} = \frac{c \lambda_{\text{min}}}{\left(\frac{3}{2} \lambda_{\text{min}}\right)^2 - \frac{\lambda_{\text{min}}}{4}} = \frac{c}{2 \lambda_{\text{min}}} = \frac{c}{\lambda_{\text{max}}} \]

\[ l_c = \frac{c}{\Delta \nu} = \lambda_{\text{max}} \]

11.2-1) \( t = \frac{2(d_1 - d_0)}{c} \), \( \Delta V = 5 \times 10^9 \text{ Hz} = \frac{3.2}{T_c} \)

For \( \frac{V}{2} \), \( t = 7 \text{ cm} \)

\[ \Delta d = d_1 - d_0 = 6.2 \mu m \]
4) For an extended source truncated by a square aperture, the source $U(r,t)$ becomes $U(r,t) * \text{rect}(\frac{r}{a})$. In the far field, this field becomes proportional to the $\text{FT}\left[U(r,t) * \text{rect}(\frac{r}{a})\right]$, which can be written as $\text{FT}[U(r,\omega)]$ convolved by $\text{sinc}(\pi ak)$. For a really wide sinc function, the center lobe acts like a low-pass filter, when convolved by $\text{FT}[U(r,\omega)]$. The sinc essentially “smooths” the Fourier transform of $U(r,t)$. This smoothing process reduces the scatter angles $k$, thereby making the function more coherent. For example, if we had a complex wave function composed of 10 different angles plane waves (each with different amplitude) and we removed 8 of the plane waves, we would have a more coherent resulting wave function. Mathematically the scatter angles, $k$, can be determined to be $\frac{2\pi}{a} = k_x = \frac{2\pi}{\lambda z} x_c$ in the far field. For a coherence area of $A_c = x_c^2 = \frac{2^2}{\Delta \Omega}$, we find a relationship of $\frac{z}{a} = 53$ for frequency $\omega = 10^{15}\text{rad/s}$ and $x_c = 1\text{mm}$. In other words, for a smaller aperture $a$, the length away from the aperture $z$ will be shorter.
5) \[ S(w) = S_1(w) + S_2(w) \]

\[ S(w) = e^{-(w-w_0)^2/2\Delta \omega_1^2} + e^{-(w-w_0)^2/2\Delta \omega_2^2} \]

\[ g_{12}(z) = \text{auto-correlation} = \int \tilde{g} \hat{S}(w)^2. \]

Using the Fourier pair

\[ \mathcal{F}\{e^{\alpha t^2/2}\} = e^{-\alpha \xi^2/2} \]

\[ g_{12}(z) = \frac{\Delta \omega_1^2}{\sqrt{2\pi}} e^{-\alpha_1 z^2/2} (\cos \omega_1 z + j \sin \omega_1 z) \]

\[ g_{12}(z) = \frac{\Delta \omega_2^2}{\sqrt{2\pi}} e^{-\alpha_2 z^2/2} (\cos \omega_2 z + j \sin \omega_2 z) \]

The auto-correlation function is the sum of two Gaussians modulated by phase terms \( e^{j \omega_1 z} \) and \( e^{j \omega_2 z} \). To understand the effect of these phase terms we can write \( g_{12}(z) \) as

\[ g_{12}(z) = A(z) e^{j \omega_1 z} + B(z) e^{j \omega_2 z} \]

\[ = e^{j (\omega_1 + \omega_2) z/2} \left[ A(z) e^{j (w_1 - w_2) z/2} + B(z) e^{-j (w_1 - w_2) z/2} \right] \]

\[ A e^{j \omega_1 t} + B e^{j \omega_2 t} = (A + B) \cos \omega t + j (A - B) \sin \omega t. \]

\[ g_{12}(z) = e^{j (\omega_1 + \omega_2) z/2} \left[ (A(z) + B(z)) \cos (w_1 - w_2) z + j (B(z) - A(z)) \sin (w_1 - w_2) z \right] \]

This implies that the Gaussians \( A(t) \) and \( B(t) \) are sinusoidally modulated. The central peak of \( g_{12}(z) \) is narrower for the composite \( S(w) \) than for individual studies \( S_1(w) \) and \( S_2(w) \) leading to better depth resolution.