Transmission through Planar Plates.
(a) Use Snell’s law to show that a ray entering a planar plate of thickness $d$ and refractive index $n_1$ (placed in air; $n \approx 1$) emerges parallel to its initial direction. The ray need not be paraxial. Derive an expression for the lateral displacement of the ray as a function of the angle of incidence $\theta$. Explain your results in terms of Fermat’s principle.
(b) If the plate instead comprises a stack of $N$ parallel layers stacked against each other with thicknesses $d_1, d_2, \ldots, d_N$ and refractive indexes $n_1, n_2, \ldots, n_N$, show that the transmitted ray is parallel to the incident ray. If $\theta_m$ is the angle of the ray in the $m$th layer, show that $n_m \sin \theta_m = \sin \theta$, $m = 1, 2, \ldots$.

Lens in Water. Determine the focal length $f$ of a biconvex lens with radii 20 cm and 30 cm and refractive index $n = 1.5$. What is the focal length when the lens is immersed in water ($n = \frac{4}{3}$)?

Numerical Aperture of a Cladless Fiber. Determine the numerical aperture and the acceptance angle of an optical fiber if the refractive index of the core is $n_1 = 1.46$ and the cladding is stripped out (replaced with air $n_2 \approx 1$).

Fiber Coupling Spheres. Tiny glass balls are often used as lenses to couple light into and out of optical fibers. The fiber end is located at a distance $f$ from the sphere. For a sphere of radius $a = 1$ mm and refractive index $n = 1.8$, determine $f$ such that a ray parallel to the optical axis at a distance $y = 0.7$ mm is focused onto the fiber, as illustrated in Fig. P1.2-10.

Extraction of Light from a High-Refractive-Index Medium. Assume that light is generated isotropically in all directions inside a material of refractive index $n = 3.7$ cut in the shape of a parallelepiped and placed in air ($n = 1$) (see Exercise 1.2-6).
(a) If a reflective material acting as a perfect mirror is coated on all sides except the front side, determine the percentage of light that may be extracted from the front side.
(b) If another transparent material of refractive index $n = 1.4$ is placed on the front side, would that help extract some of the trapped light?
symmetry about the \( z \) axis and refractive index \( n(\rho), \rho = \sqrt{x^2 + y^2} \). Let \( (\rho, \phi, z) \) be the position vector in a cylindrical coordinate system. Rewrite the paraxial ray equations, (1.3-4), in a cylindrical system and derive differential equations for \( \rho \) and \( \phi \) as functions of \( z \).

1.4-8 **Ray-Transfer Matrix of a Lens System.** Determine the ray-transfer matrix for an optical system made of a thin convex lens of focal length \( f \) and a thin concave lens of focal length \(-f\) separated by a distance \( f \). Discuss the imaging properties of this composite lens.

1.4-9 **Ray-Transfer Matrix of a GRIN Plate.** Determine the ray-transfer matrix of a SELFOC plate [i.e., a graded-index material with parabolic refractive index \( n(y) \approx n_0(1 - \frac{1}{2} \alpha^2 y^2) \)] of thickness \( d \).

1.4-10 **The GRIN Plate as a Periodic System.** Consider the trajectories of paraxial rays inside a SELFOC plate normal to the \( z \) axis. This system may be regarded as a periodic system made of a sequence of identical contiguous plates, each of thickness \( d \). Using the result of Prob. 1.4-9, determine the stability condition of the ray trajectory. Is this condition dependent on the choice of \( d \)?

1.4-11 **Recurrence Relation for a Planar-Mirror Resonator.** Consider a planar-mirror optical resonator, with mirror separation \( d \), as a periodic optical system. Determine the unit ray-transfer matrix for this system, demonstrating that \( b = 1 \) and \( F = 1 \). Show that there is then only a single root to the quadratic equation (1.4-27) so that the ray position must then take the form \( \alpha + m\beta \), where \( \alpha \) and \( \beta \) are constants.

1.4-12 **4 \times 4 Ray-Transfer Matrix for Skewed Rays.** Matrix methods may be generalized to describe skewed paraxial rays in circularly symmetric systems, and to astigmatic (non-circularly symmetric) systems. A ray crossing the plane \( z = 0 \) is generally characterized by four variables — the coordinates \( (x, y) \) of its position in the plane, and the angles \( (\theta_x, \theta_y) \) that its projections in the \( x-z \) and \( y-z \) planes make with the \( z \) axis. The emerging ray is also characterized by four variables linearly related to the initial four variables. The optical system may then be characterized completely, within the paraxial approximation, by a \( 4 \times 4 \) matrix.

(a) Determine the \( 4 \times 4 \) ray-transfer matrix of a distance \( d \) in free space.

(b) Determine the \( 4 \times 4 \) ray-transfer matrix of a thin cylindrical lens with focal length \( f \) oriented in the \( y \) direction. The cylindrical lens has focal length \( f \) for rays in the \( y-z \) plane, and no focusing power for rays in the \( x-z \) plane.